

# **Theoretical Approaches for the Analyses of Scanning Probe Microscopy**

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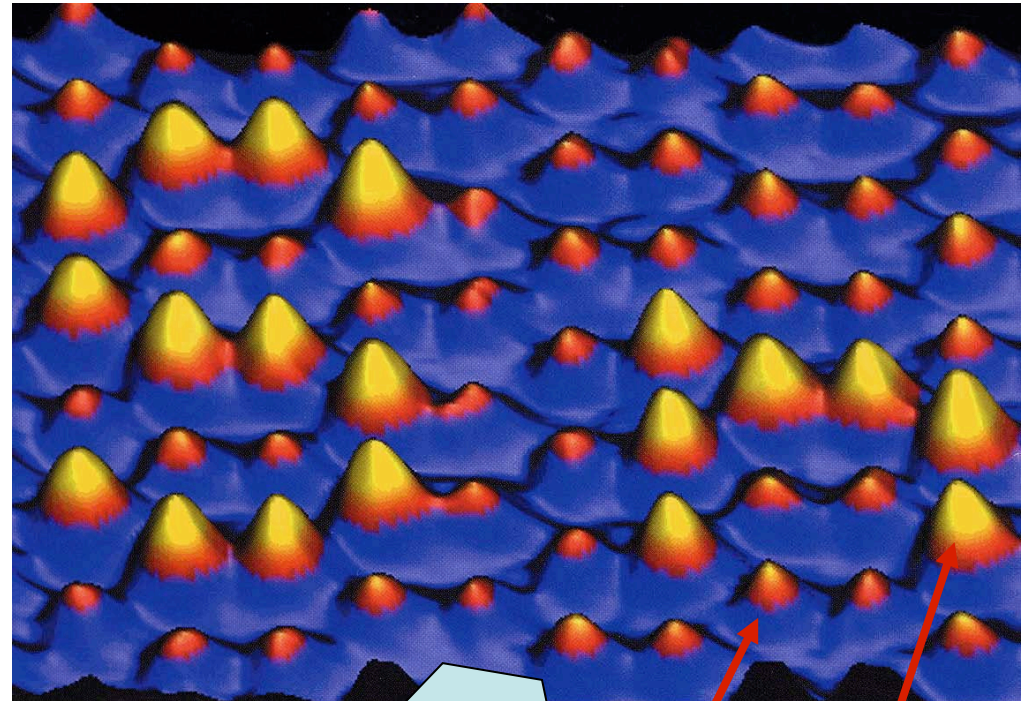
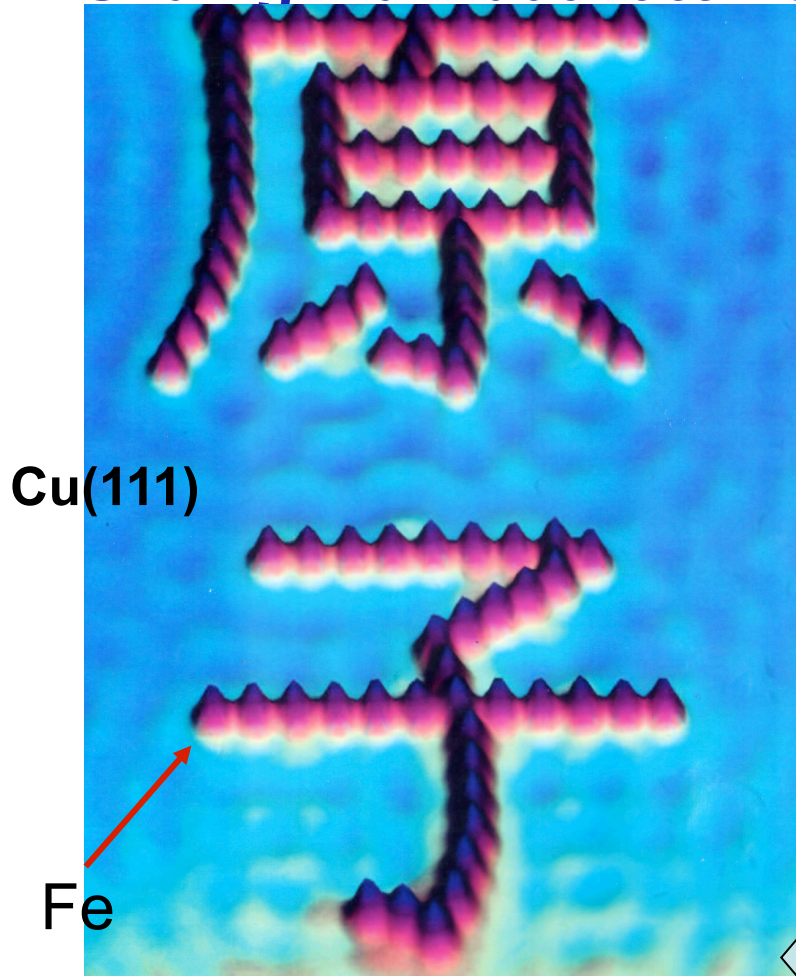
# outline

- 1 Introduction
- 2 Theoretical methods for STM -1
  - Bardeen's Perturbation approach
  - Effect of the tip, some case studies of the results
- 3 Theoretical methods for STM -2
  - Non-equilibrium Green's Function and other methods
- 4 Coherent and Dissipating Tunnelling
  - tunneling current distribution, features of the resonant tunneling
  - energy loss spectral function, zero bias anomaly
- 5 Theory of dynamic AFM -1 in vacuum
  - derivation of the harmonic oscillator model,
  - dissipation image, tip effects, some case studies
- 6 Fast simulation methods
- 7 Nano-mechanics of protein molecules
- 8 Theory of dynamic AFM -2 in liquids
  - cantilever oscillation, oscillatory hydration force,
  - force mediated by water, case studies
- 9 Summary and outlook

# The letters written by atoms with SPM tip

Sliding individual atoms

Interchanging atoms

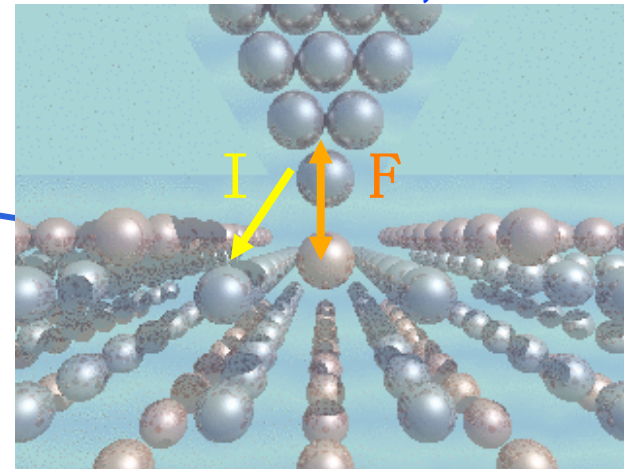
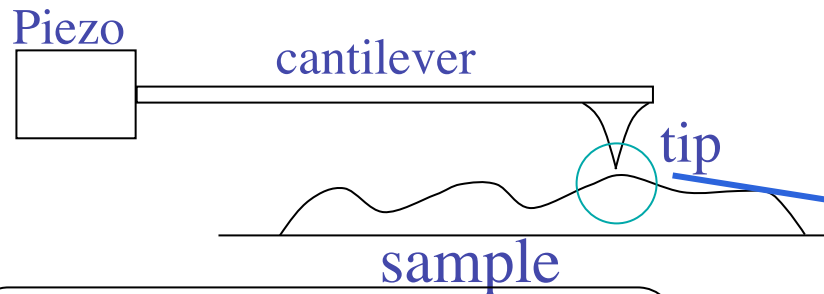


Letters with Fe atoms on Cu(111), D.Eigler  
1993

Y.Sugimoto, M.Abe, S.Hirayama,  
N.Oyabu, O.Custance and  
S.Morita, Nature Mterials 4 (2005)  
156

# Theory of Scanning Probe Microscopy

STM/STS, AFM  
(contact, noncontact)  
KFM, SNOM



**What and how**

Does SPM see the sample?

Effect of the tip atomic structure/  
Atom kind?

Information transfer  
to macro-system

Formation and  
control of  
nanostructures

Observation of quantum  
Phenomena and  
functionalization

Atomic scale Observables

Force? Displacement?

Fluctuations and dissipations

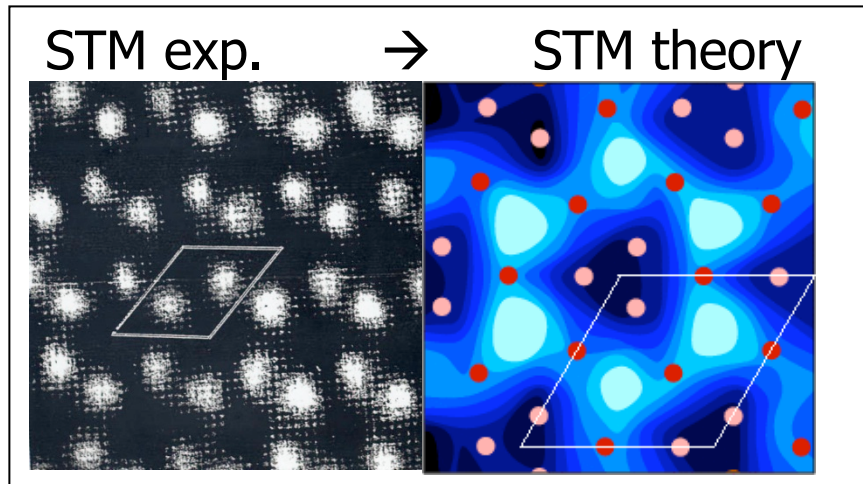
Influence of wave functions

Quantum Coherence and de-  
coherence

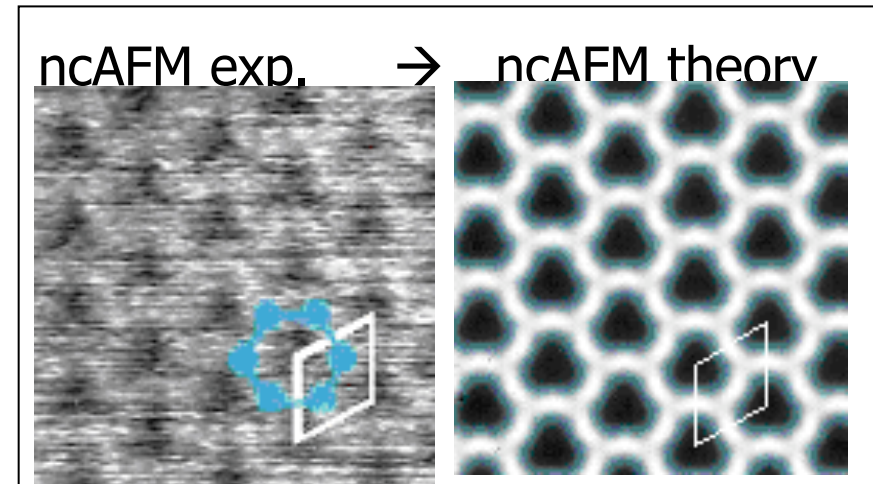
# What can be seen by SPM ?

From examples of the first-principles simulation

Si(111) $\sqrt{3} \times \sqrt{3}$ -Ag surface



S. Watanabe, M. Tsukada, Phys. Rev. B. 1991



N. Sasaki, S. Watanabe, M. Tsukada, Phys. Rev. Lett. 2002

**Remarkably different images for the same surface by STM and ncAFM!**

Bright spots in the STM image does not correspond atoms

→ Importance of quantum mechanical effect

Large temperature dependence of ncAFM images

→ reproduced by theoretical calculation

**→ Very important role of Simulation!**



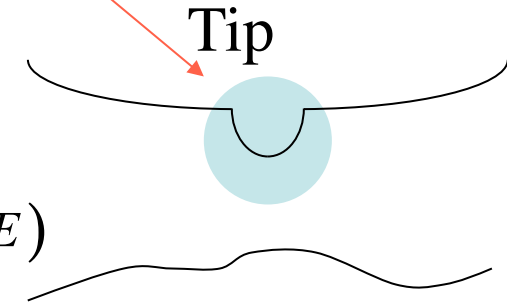
# The relation to LDOS

$$I(\mathbf{R}) = \frac{2\pi e}{\hbar} \int_{E_F}^{E_F + eV} A(\mathbf{R}, E, E - eV) dE$$

$$A(\mathbf{R}, E, E') = \sum_{v \in \text{Tip}} \int_{\text{Tip}} d\mathbf{r} d\mathbf{r}' V_T(\mathbf{r}) V_T(\mathbf{r}') \psi_v(\mathbf{r}') \psi_v^*(\mathbf{r}) \delta(E' - E_v) \times \tilde{G}^S(\mathbf{r} + \mathbf{R}, \mathbf{r}' + \mathbf{R}; E) \gamma(z; E) \gamma(z'; E)$$

$$\tilde{G}^S = \frac{G^S}{\gamma(z; E) \gamma(z'; E)} \quad \gamma(z; E) = \exp\left(-z \frac{\sqrt{2m|E|}}{\hbar}\right)$$

$W(\mathbf{r}, \mathbf{r}'; E')$  → weight function



Moment expansion

Surface DOS

$$A(\mathbf{R}, E, E') = \sum_{v \in \text{Tip}} \left| \int_{\text{tip}} d\mathbf{r} V_T(\mathbf{r}) \psi_v(\mathbf{r}) \gamma(z, E) \right|^2 \delta(E' - E_v) \times \rho(\mathbf{R}, E) + \sum_{m \geq 1} \sum_{n \geq 1} \mu_{mn}(E') \nabla \dots \nabla \nabla' \dots \nabla' \tilde{G}^S(\mathbf{r}, \mathbf{r}'; E) \Big|_{\mathbf{r}' = \mathbf{R}, \mathbf{r} = \mathbf{R}}$$

Sample surface

Moment of the weight function

$$\mu_{mn}(E') = \int \mathbf{r} \dots \mathbf{r} \mathbf{r}' \dots \mathbf{r}' W(\mathbf{r}, \mathbf{r}') d\mathbf{r} d\mathbf{r}'$$

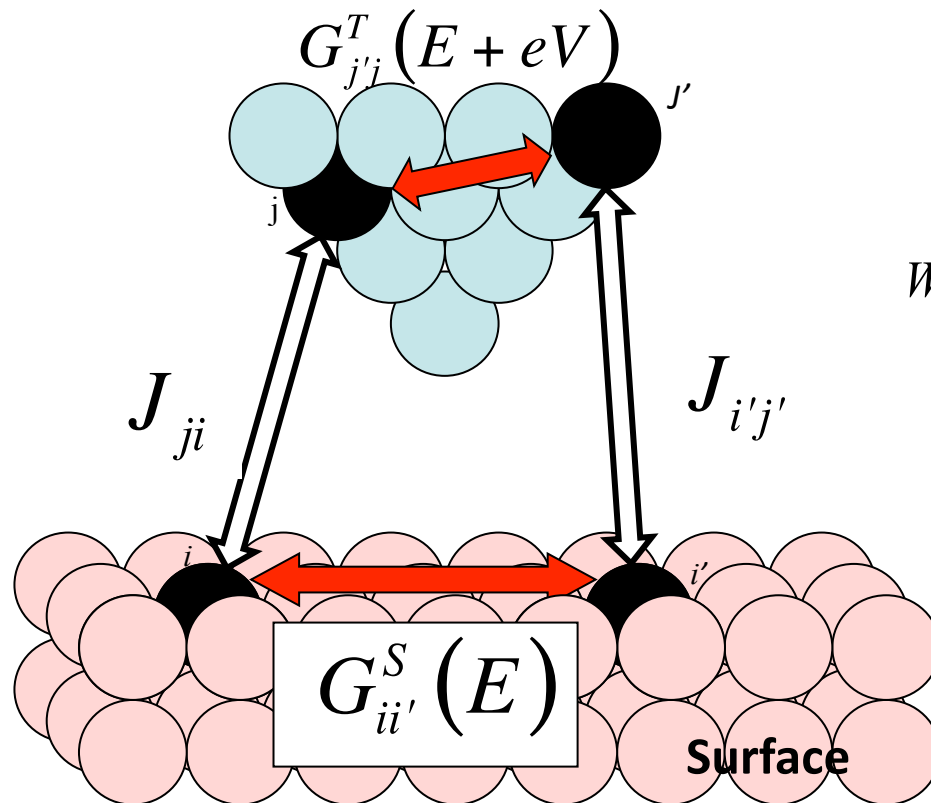
... Tersoff-Hamann

Taking only the first term,  $\longrightarrow I(\mathbf{R}) \mu_{r_{\text{surface}}}(\mathbf{R}, E_F) V$

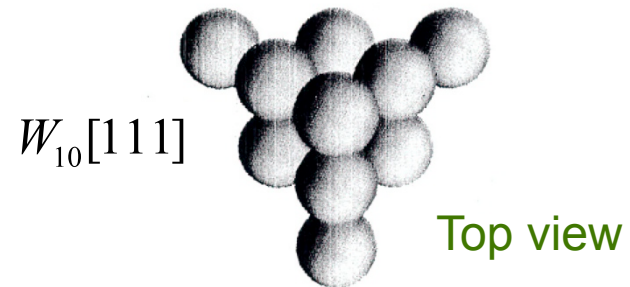
# Tunneling current in the LCAO representation

Tunneling current at the tip position  $\mathbf{R}$  and for the bias  $V$  is expressed as

$$I(\mathbf{R}, V) = \frac{2\pi e}{\hbar} \int_{E_F^L}^{E_F^R} \sum_{ii'jj'} G_{ii'}^S(E) J_{ij'}(\mathbf{R}) G_{jj'}^T(E + eV) J_{ji}(\mathbf{R}) dE$$

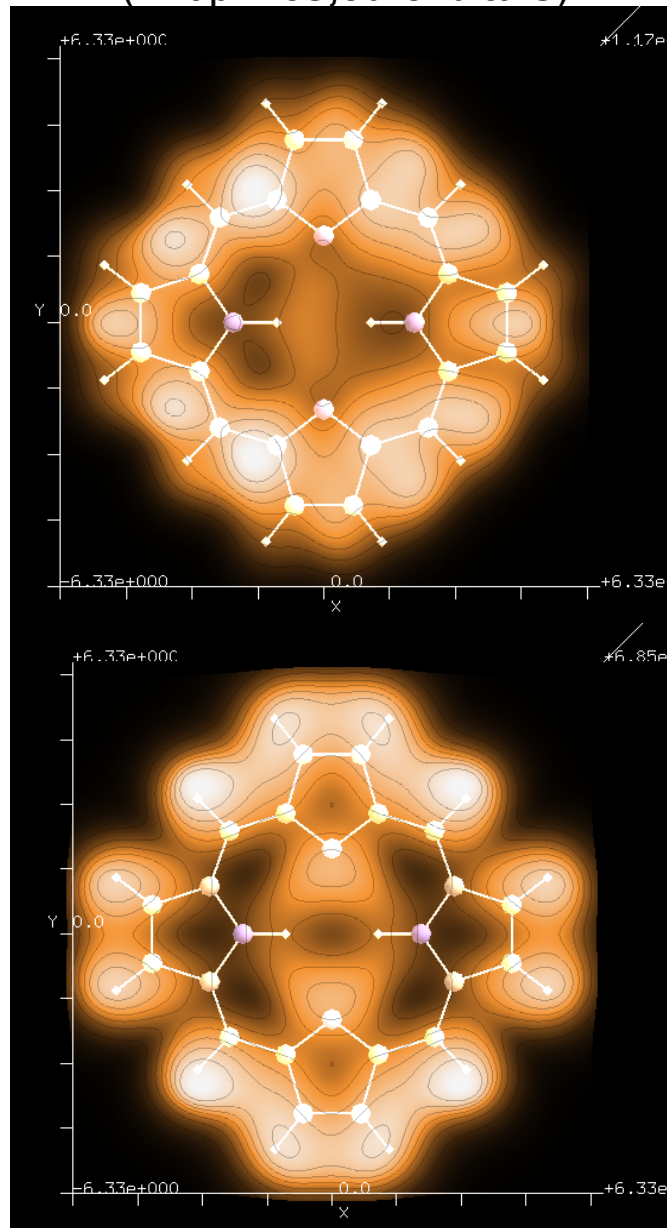


Cluster models for the tip





# STM image of porphyrin (W tip : 6s,5d orbitals)



(W tip: 6s orbital)

# Simulation of STM image

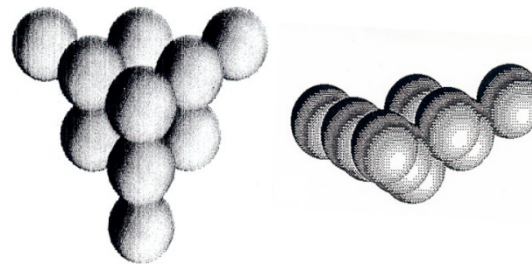
Energy density of  
Tunnel current

$$I(\mathbf{R}, V) \propto \sum_{ij} G_{ii}^S(E) J_{ij} G_{jj}^T(E + eV) J_{ji}$$

Green Function

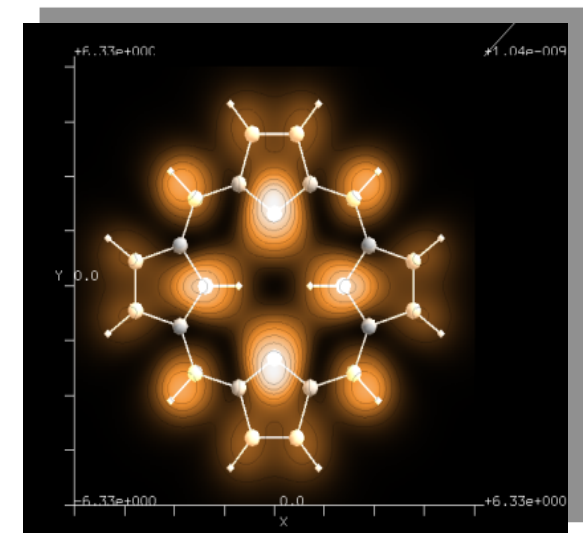
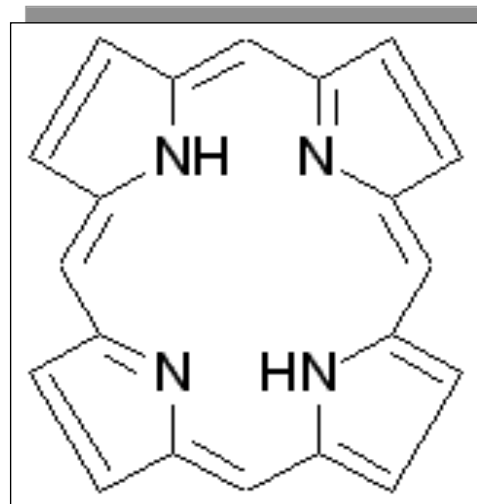
Tunnel matrix element

W<sub>10</sub>[111] tip model

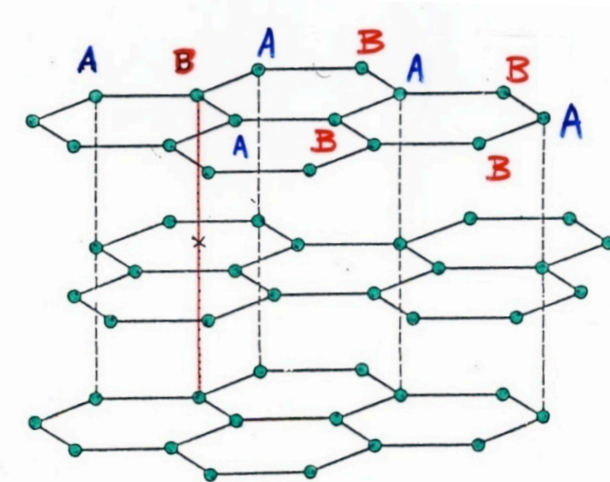
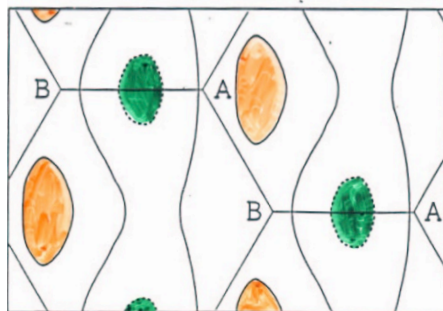
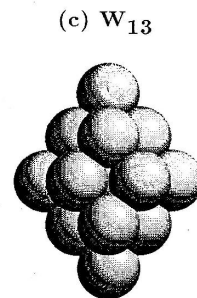
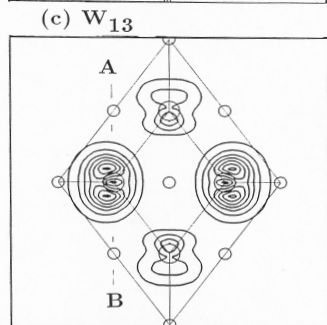
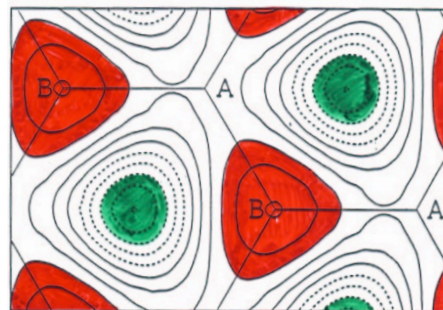
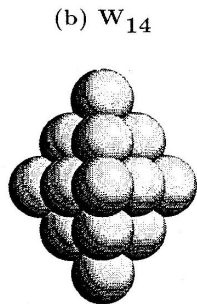
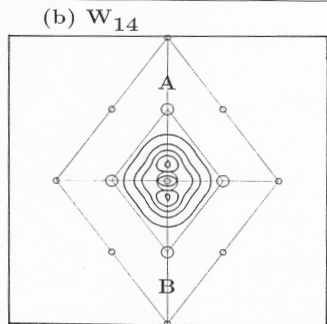
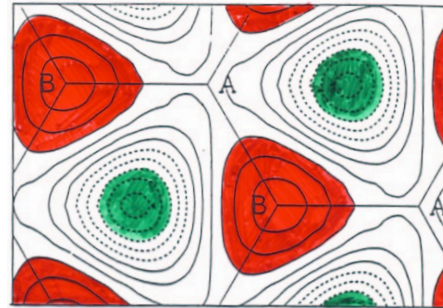
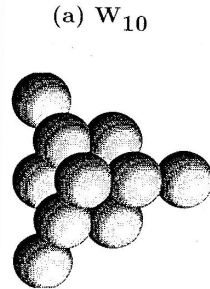
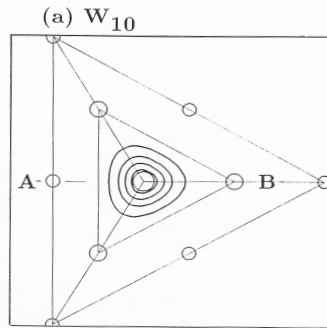


DVX $\alpha$   
calculation

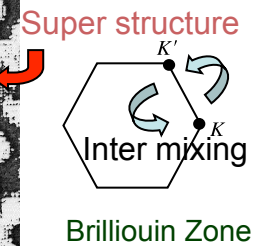
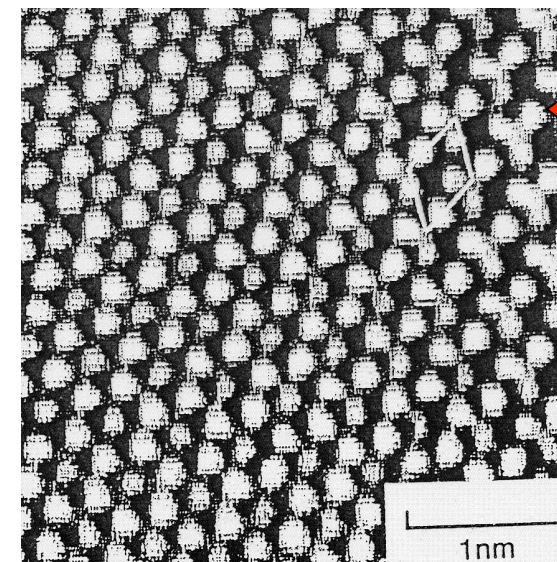
LDOS of molecule



# Simulation of the STM image of graphite



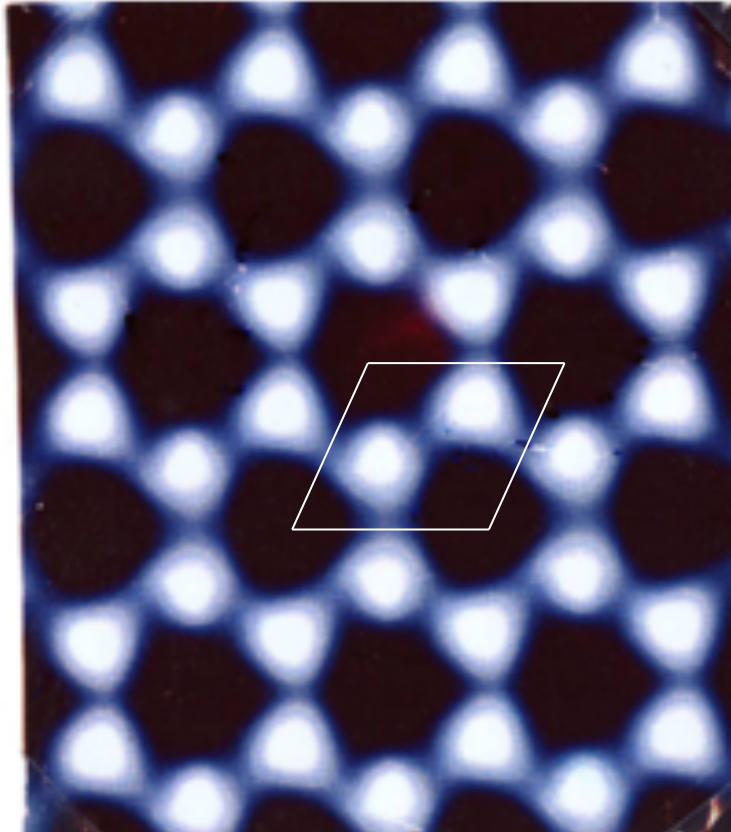
Nakagawa et al, Proc. Ann. Meetingd of  
The Phys. Soc. Jpn, (1989) 374



Isshiki, Kobayashi, Tsukada  
J. Vac. Sci. Technol, B9(2)(1991)475

# Si(111) $\sqrt{3}\times\sqrt{3}$ -Ag 表面のSTM像 実験と理論

Theoretical Image for HCT model



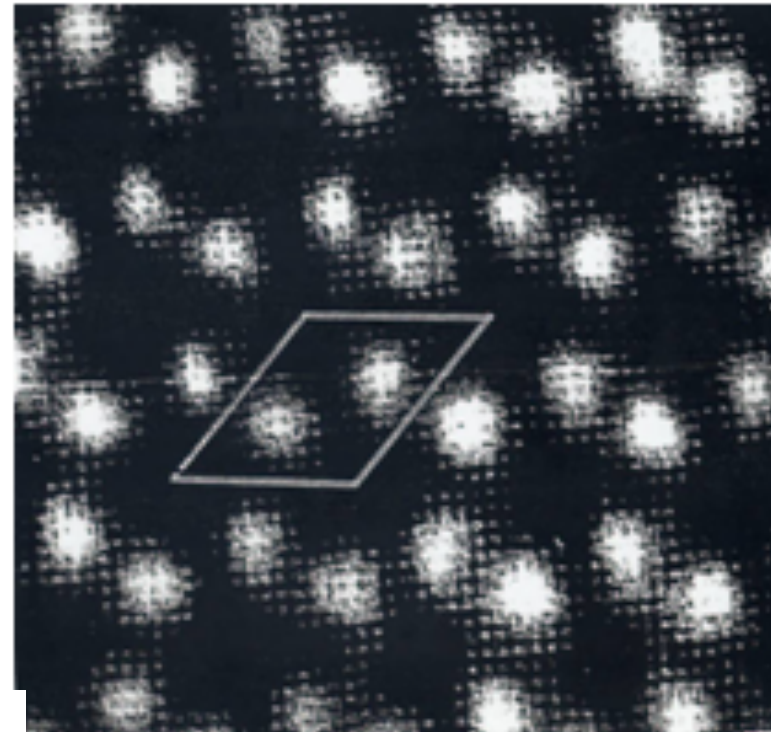
W [111] tip, tip height 3.6Å

S.Watanabe, M.Aono and M.Tsukada  
Phys. Rev. B44 ('91) 8330

Positive surface bias about 2eV

E.J.van Loenen, et al, PRL  
58 (1987) 373

Experiment

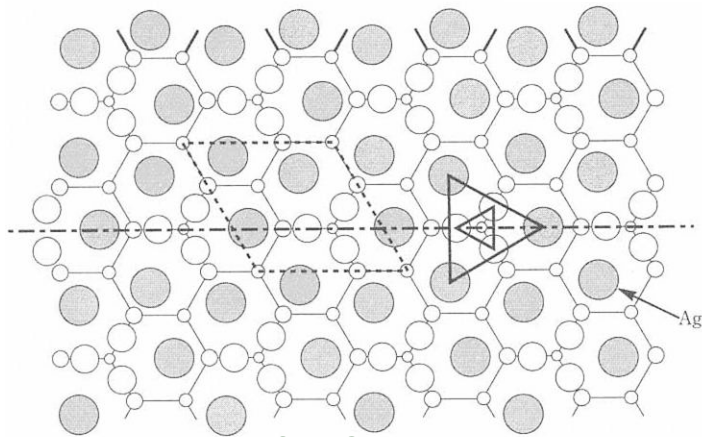
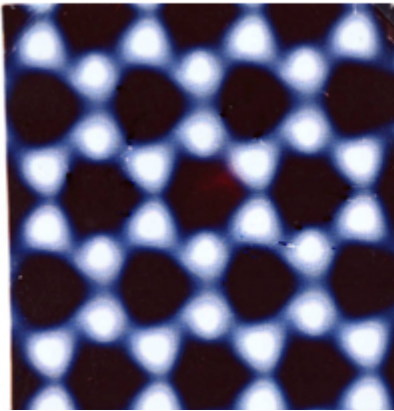


# Si(111) $\sqrt{3}\times\sqrt{3}$ -Ag Surface(HCT model)

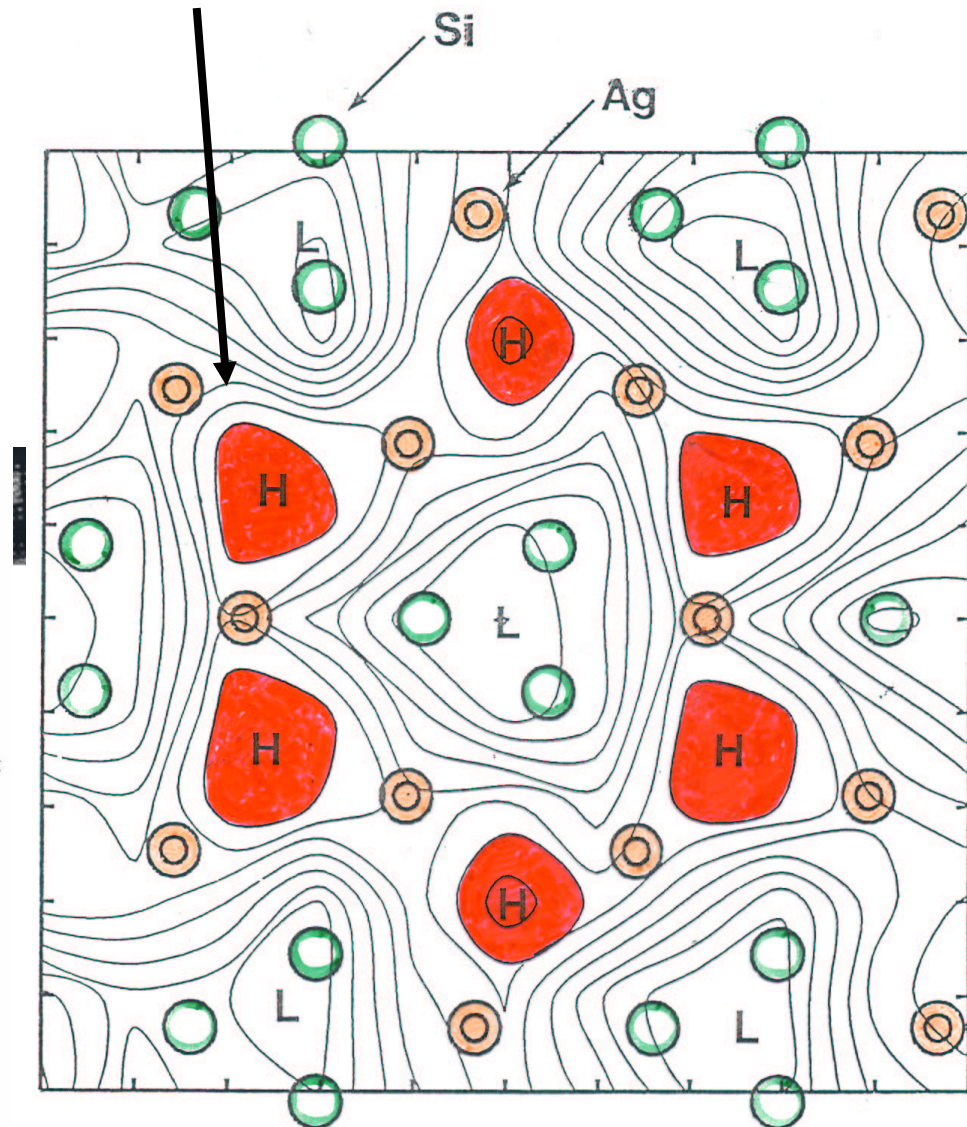
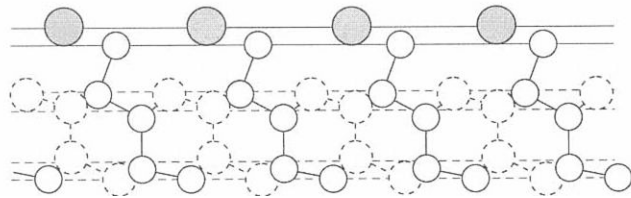
Wartanabe, Aono, Tsukada(1991) Phys.Rev.B44 (1991)8330

Bright spot located in the Ag triangle

Theory HCT model



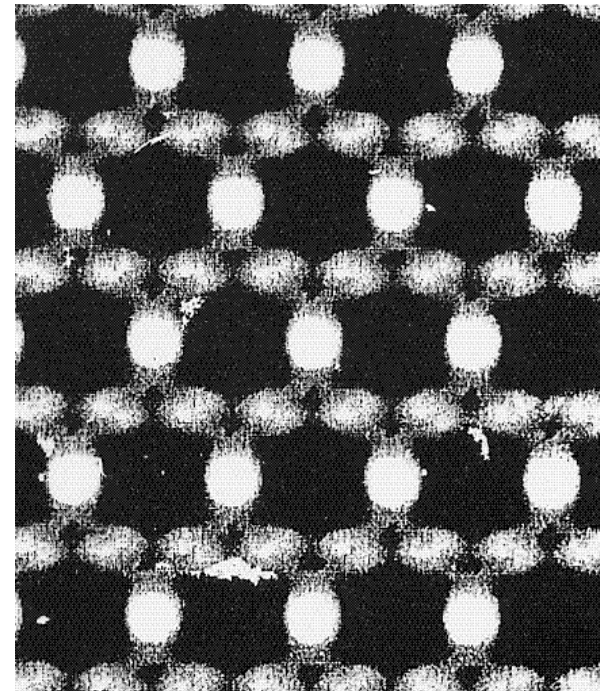
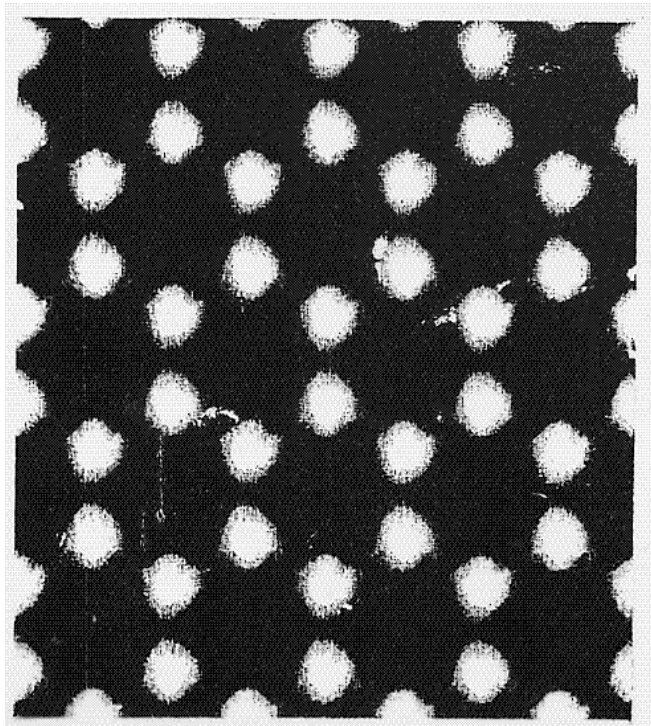
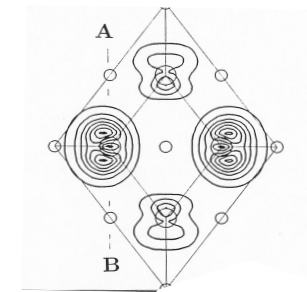
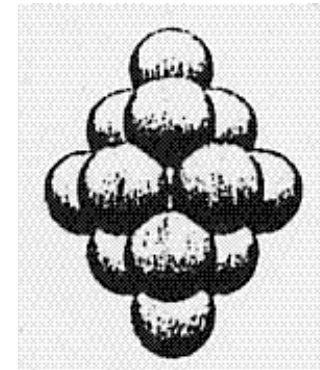
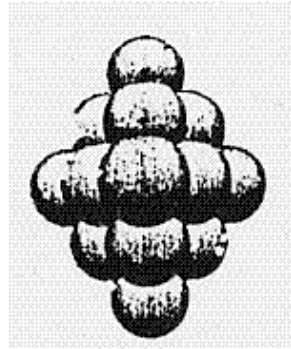
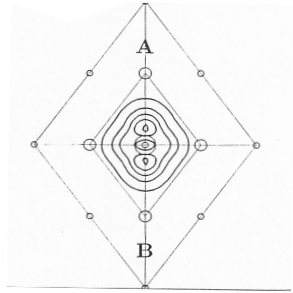
Actual structure of HCT model



# Effect of Tip Shape

If the apex atom is removed

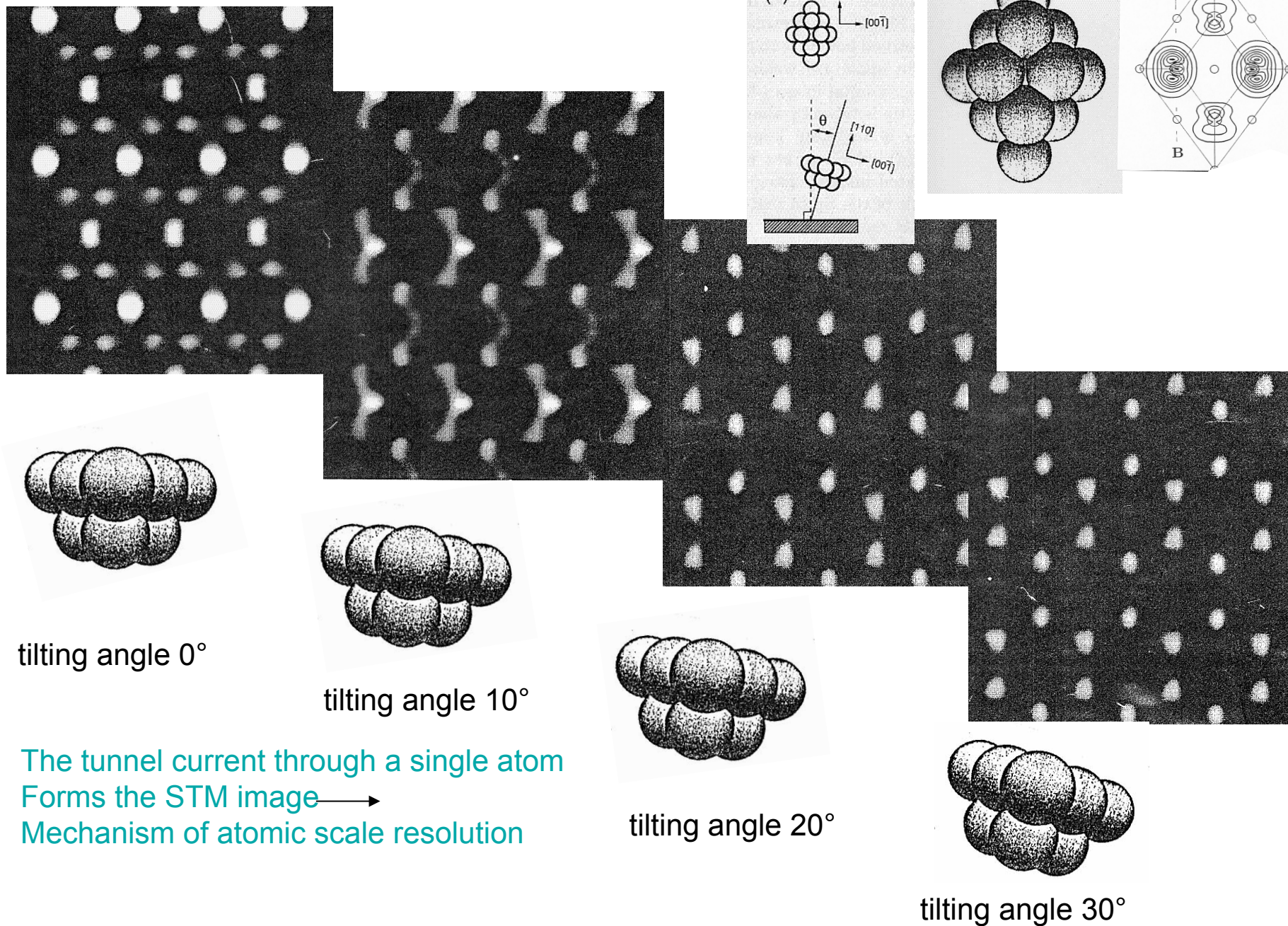
W tip [110]



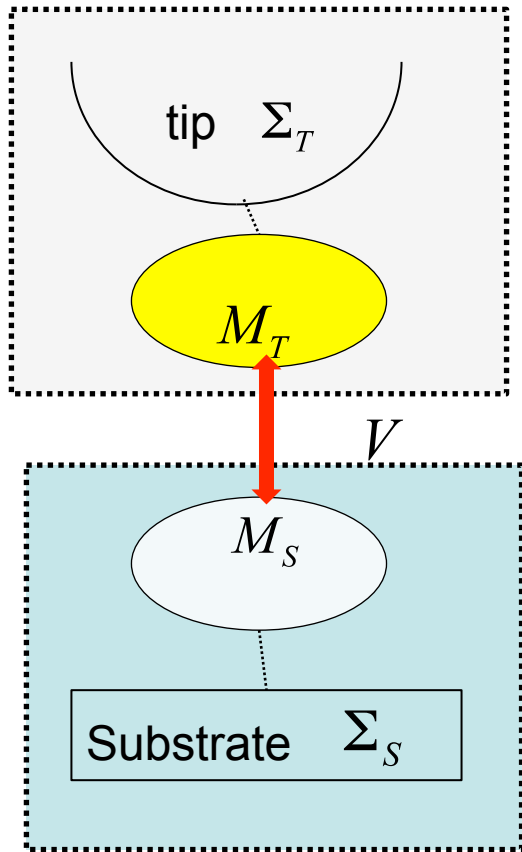
**Abnormal image is obtained!**

S.Watanabe, M.Aono and M.Tsukada, Jpn. J. Appl. Phys., 32 ('93) 2911

# Effect of the tip tilting on the STM images



# Tunnel current by a molecule adsorbed tip for molecular samples



## Non-equilibrium Green's Function theory - beyond Bardeen's Approximation

Green's function for the tip side

$$g^T = (E - i\eta - H_{M_T} - \Sigma_T)^{-1}$$

Green's function for the total system

$$G_{ip}^{R/A} = \langle i | g^T V g^S + g^T V g^S V g^T V g^S + \dots | p \rangle$$

Site of  $M_T$ 
Site of  $M_S$

$$= \langle i | g^T \Lambda g^S | p \rangle$$

Total interaction between the tip and the sample

$$\Lambda = V + V g^S V g^T V + V (g^S V g^T V)^2 + \dots$$

$$= V (1 - g^S V g^T V)^{-1} \approx V$$

Tunnel current

$$I = \frac{2e}{h} \int (f_T - f_S) \bar{T}_{ip}(E) dE$$

Green's function for the sample side

$$g^S = (E - i\eta - H_{M_S} - \Sigma_S)^{-1}$$

$$G_{ilp} = i(S_{ilp}^R - S_{ilp}^A) = t_{ilp}^+ \hat{g} (g_{ilp}^R - g_{ilp}^A) t_{ilp}^-$$

$$g^{R/A} = (E - i\eta - H_{M_T} - H_{M_S} - \Sigma_S^{R/A} - \Sigma_T^{R/A})^{-1}$$

$$\bar{T}_{ip} = Tr[\Gamma_i G^R \Gamma_p G^A] = Tr[\Gamma_i g^T \Lambda g^S \Gamma_p \bar{g}^S \Lambda \bar{g}^T]$$

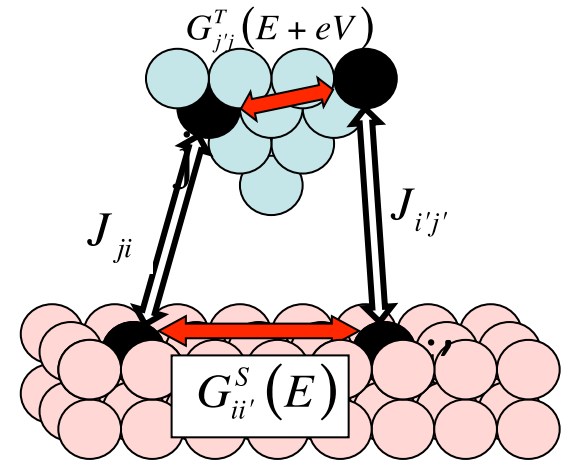
$$\approx Tr[\Gamma_i g^T V g^S \Gamma_p \bar{g}^S V \bar{g}^T] = Tr[\bar{g}^T \Gamma_i g^T V g^S \Gamma_p \bar{g}^S V]$$

$$\rightarrow Tr[G^T V G^S V]$$

# Correspondence to Bardeen's Approximation with cluster model approximation

$$I(\mathbf{R}) = \frac{2\pi e}{\hbar} \int_{E_F}^{E_F + eV} A(\mathbf{R}, E, E - eV) dE$$

$$= \frac{2\pi e}{\hbar} \int_{E_F^L}^{E_F^R} \sum_{\ddot{u}\ddot{j}\ddot{j}} G_{\ddot{u}\ddot{i}'}^S(E) J_{\ddot{i}\ddot{j}'}(\mathbf{R}) G_{\ddot{j}\ddot{j}}^T(E + eV) J_{\ddot{j}\ddot{i}}(\mathbf{R}) dE$$



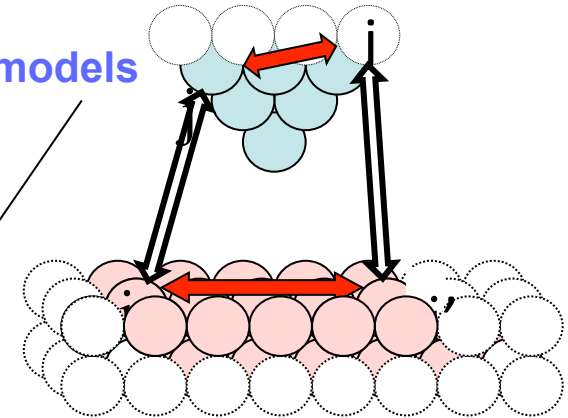
With LCAO representation  $f_n^T(\mathbf{r}) = \mathring{a}_i C_{ni}^T j_i^T(\mathbf{r})$

$$f_m^S(\mathbf{r}) = \mathring{a}_j C_{mj}^S j_j^S(\mathbf{r})$$

$$G_{j\phi}^T(E\phi) = \mathring{a}_n C_{nj\phi}^{T*} C_{nj}^T d(E\phi - E_n) @ \mathring{a}_n C_{nj\phi}^{T*} C_{nj}^T \frac{G/p}{(E\phi - E_n)^2 + G^2}$$

$$G_{\ddot{u}\ddot{i}\phi}^S(E) = \mathring{a}_m C_{m\phi}^{S*} C_{m\phi}^S d(E - E_m) @ \mathring{a}_m C_{m\phi}^{S*} C_{m\phi}^S \frac{G/p}{(E - E_m)^2 + G^2}$$

Using cluster models

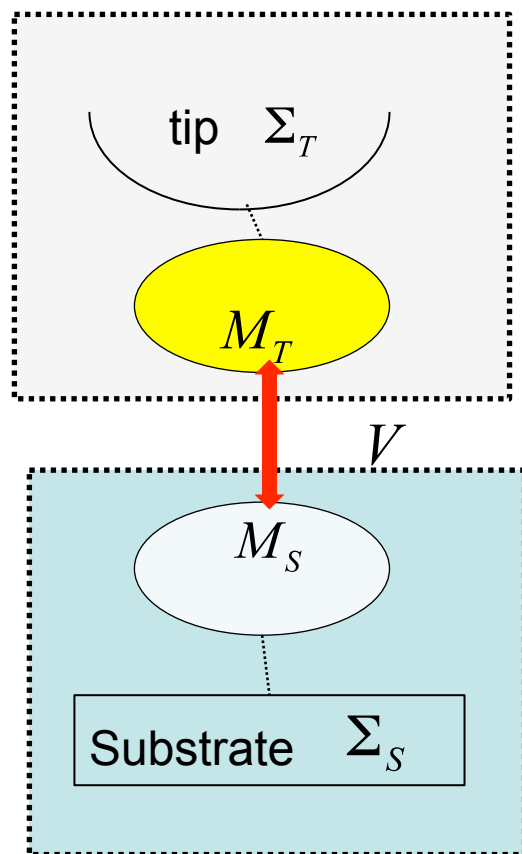


This is equivalent to the new NEGF theory

$$\bar{T}_{ip} = Tr [\bar{g}^T \Gamma_i g^T V g^S \Gamma_p \bar{g}^S V] = \mathring{a}_{ln} \frac{|\langle i|l\rangle|^2 V_{ln} |\langle n|p\rangle|^2 V_{nl} S_l^T S_n^S}{\{(E - e_l^T)^2 + (S_l^T)^2\} \{(E - e_n^S)^2 + (S_n^S)^2\}}$$



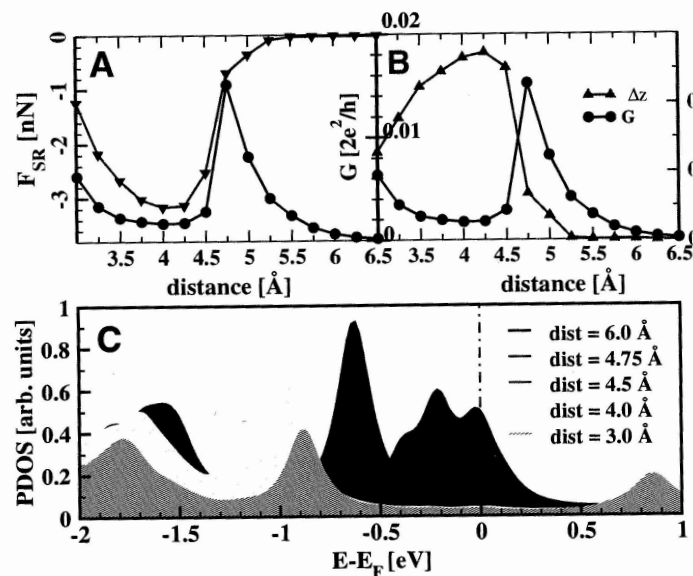
# Reduction of tunnel current with the increase of the tip-sample interaction



$$\Lambda = V + Vg^S Vg^T V + V \left( g^S Vg^T V \right)^2 + \dots$$

$$= V \left( 1 - g^S Vg^T V \right)^{-1} \approx V$$

$$I = \dot{\Phi}_T - \dot{\Phi}_S = (f_T - f_S) \text{tr} \left[ \hat{G}_T \frac{g^T Vg^S}{1 - g^S Vg^T V} \hat{G}_S \frac{\bar{g}^S V\bar{g}^T}{1 - \bar{g}^T V\bar{g}^S V} \right] \dot{\Phi}_E$$

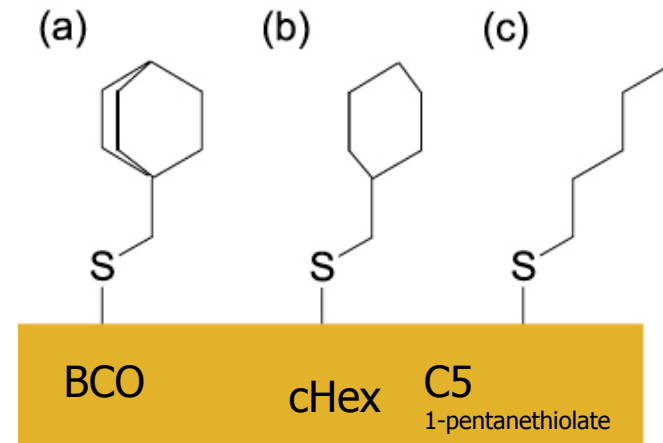
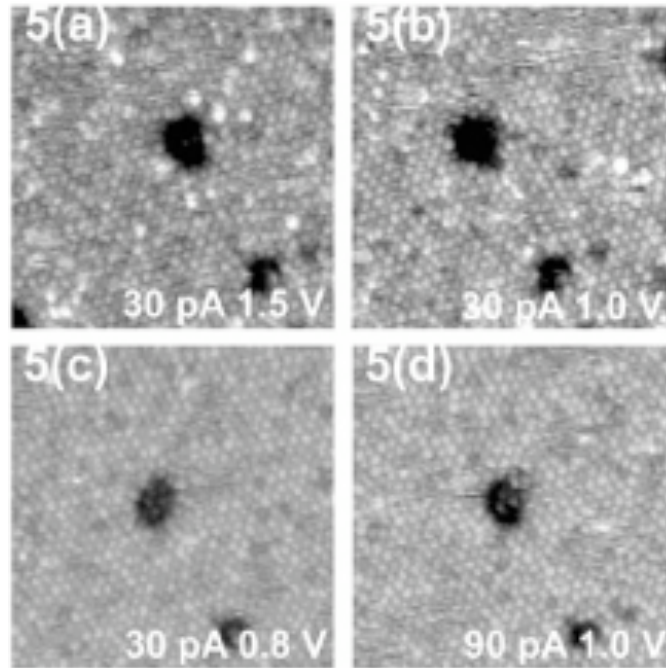


Jelinek et al,  
Phys. Rev. Lett., 101 (2008)178101

# Conduction Switching of Alkane Molecule on Au(111) by Conformation Change

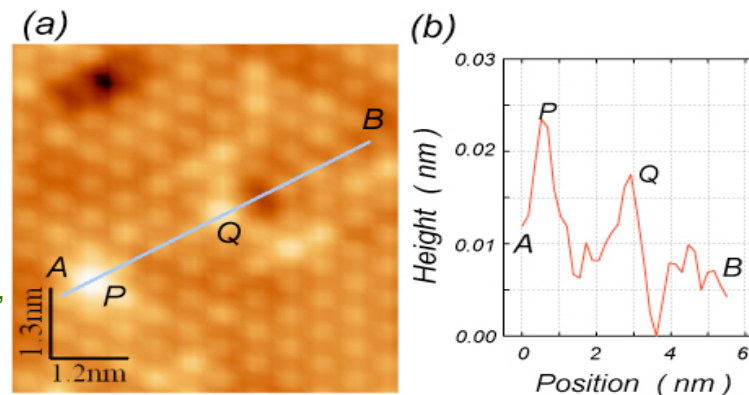
STM images

M. Suzuki, et al.,  
Nanotechnology 15,  
S150 (2004).



AFM image  
Theoretical  
simulation

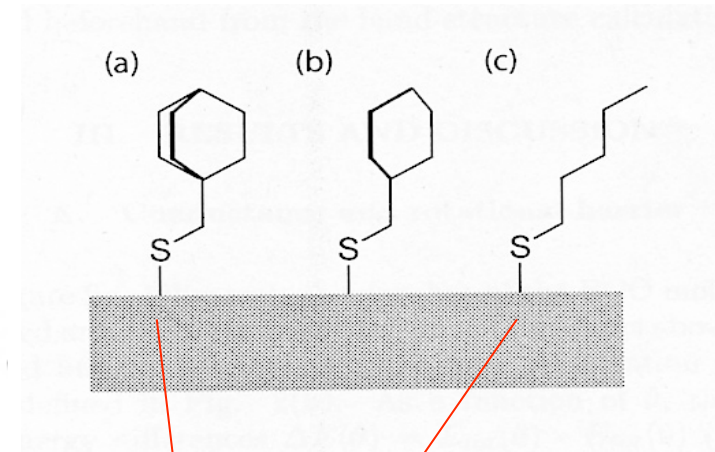
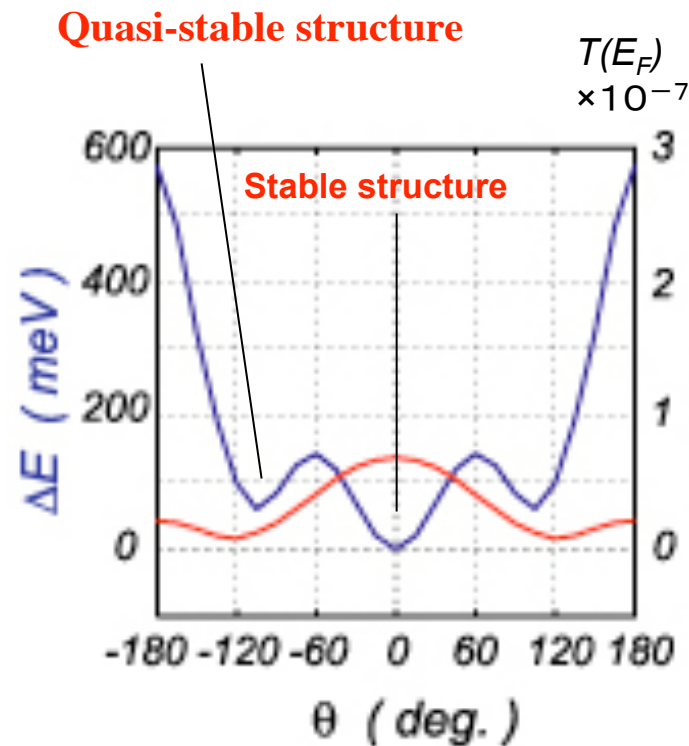
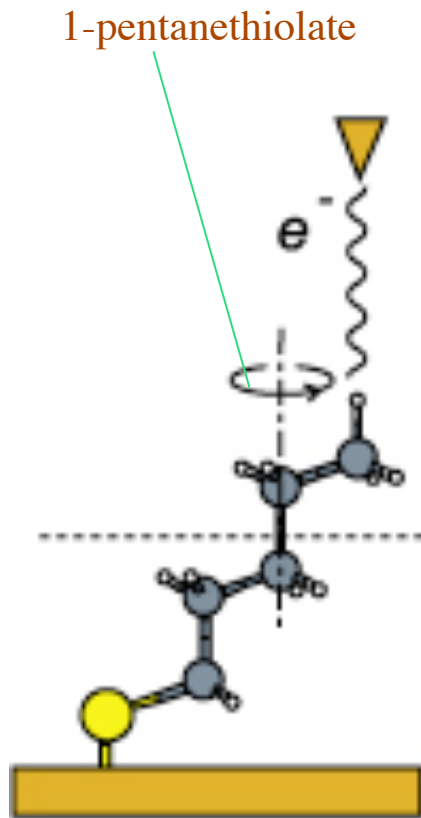
K.Tagami and M.Tsukada,  
E-J. Surf. Sci. and Nanotech.,  
4 (2006)299



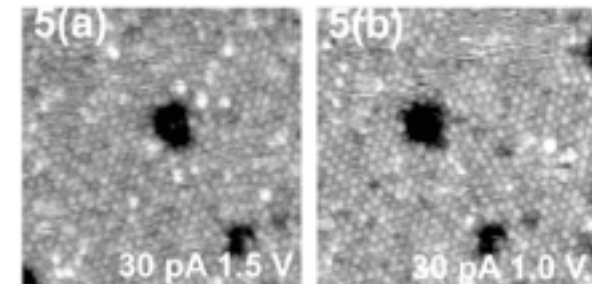
**C5** molecules embedded in  
BCO-SAM Membrane are  
observed as **bright spots**.

**They are  
blinking!**

# Conduction Switching of Alkane Molecule by Conformation change on Au(111)



1-pentanethiolate  
Bicyclo[2,2,2]octylmethylthiolate



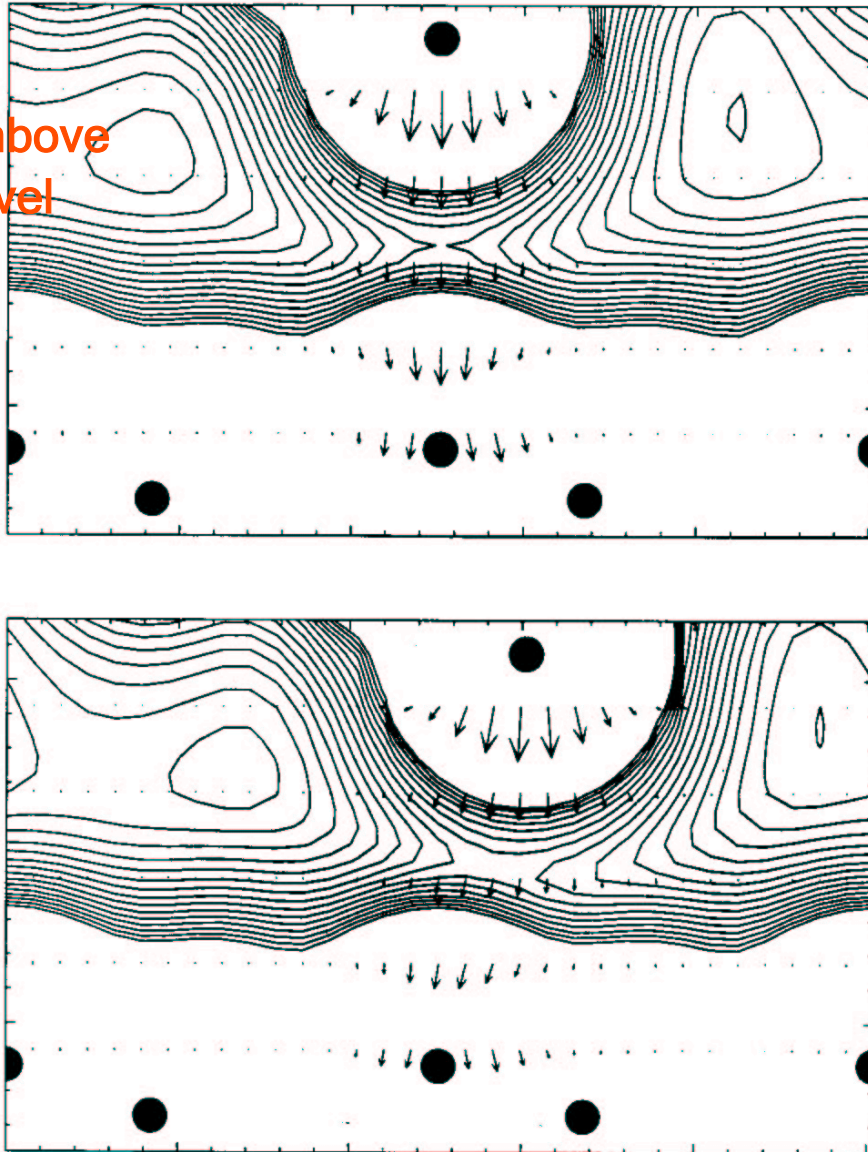
**Bright stable structure**  
**And dark quasi-stable structure appears!**

K.Tagami and M.Tsukada, *e-J. Surface Sci. and Nanotech.*, 2 ('04) 186

Coherent Tunneling/Transport  
vs  
Dissipating Tunneling

# Tunnel Current Density and Barrier

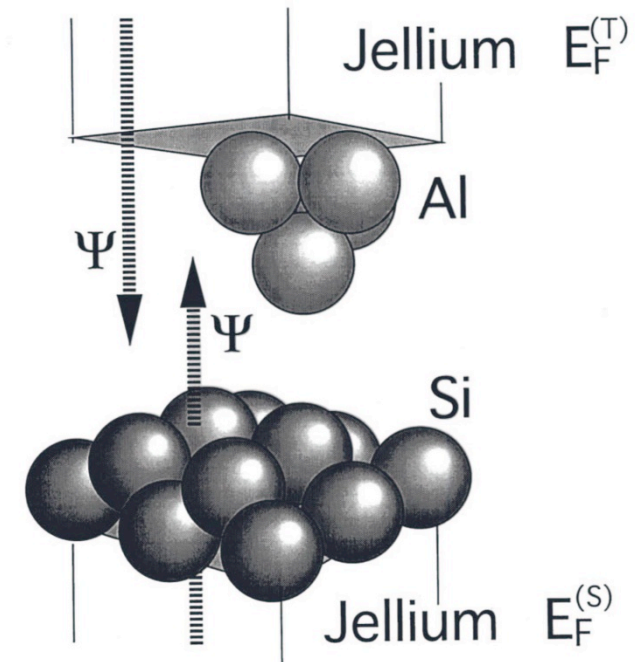
Barrier above  
Fermi level



$d=12\text{au}$   
 $V_s=2\text{V}$

Calculation with  
First principles RTM

Current is confined  
In a narrow channel !!

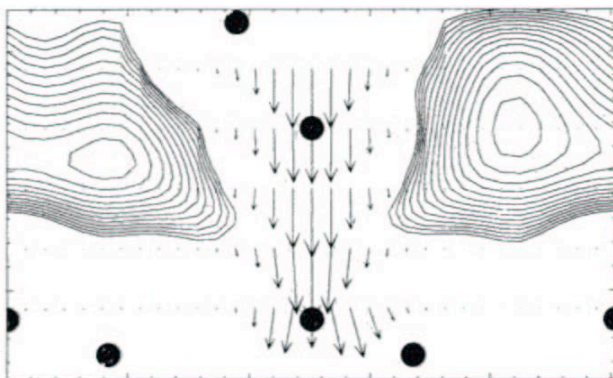


N.Kobayashi, K.Hirose and M.Tsukada,  
J.J.Appl. Phys. 35(1996)3710

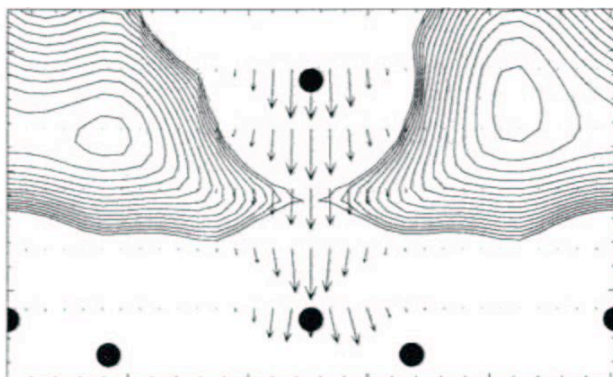
what happens at the atomic contact ?

$V_s=2V$

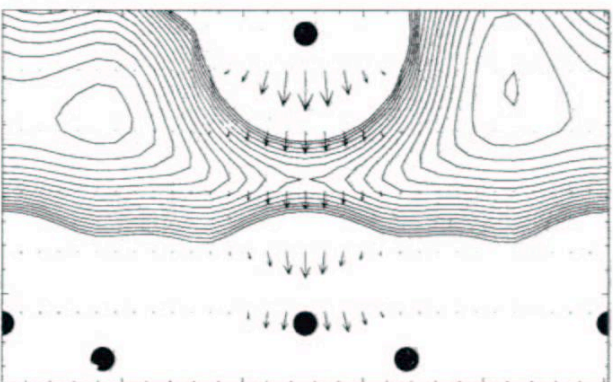
$d=8\text{au}$



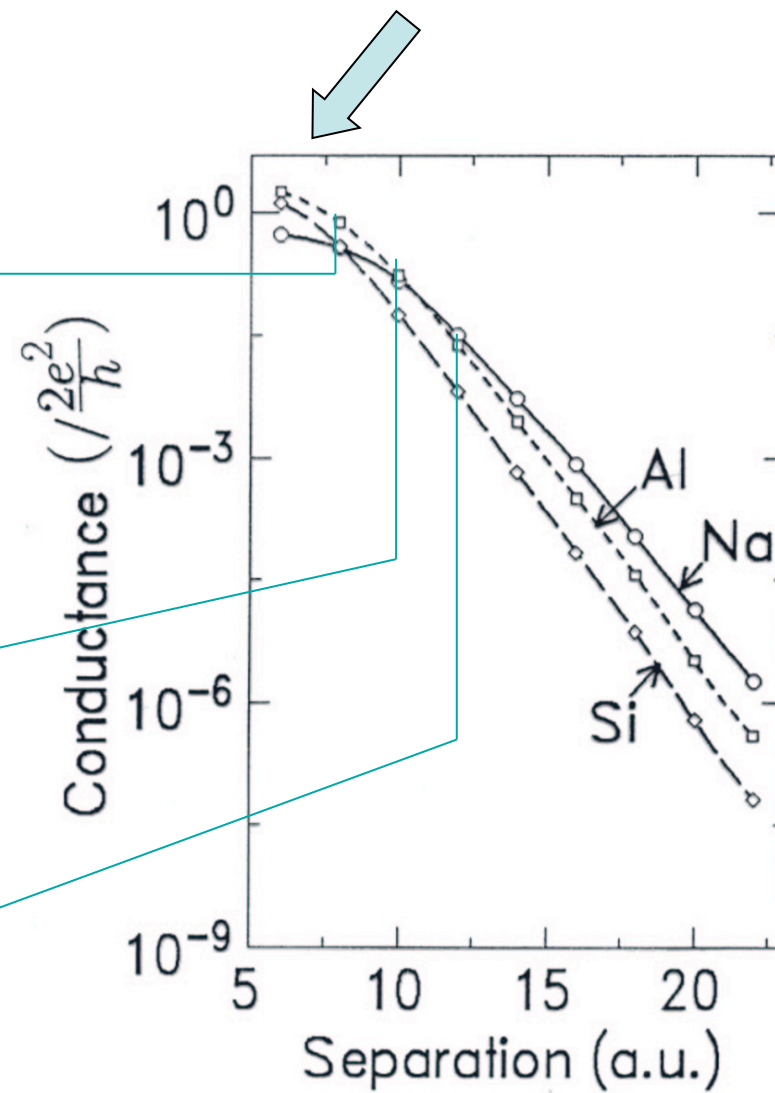
$d=10\text{au}$



$d=12\text{au}$

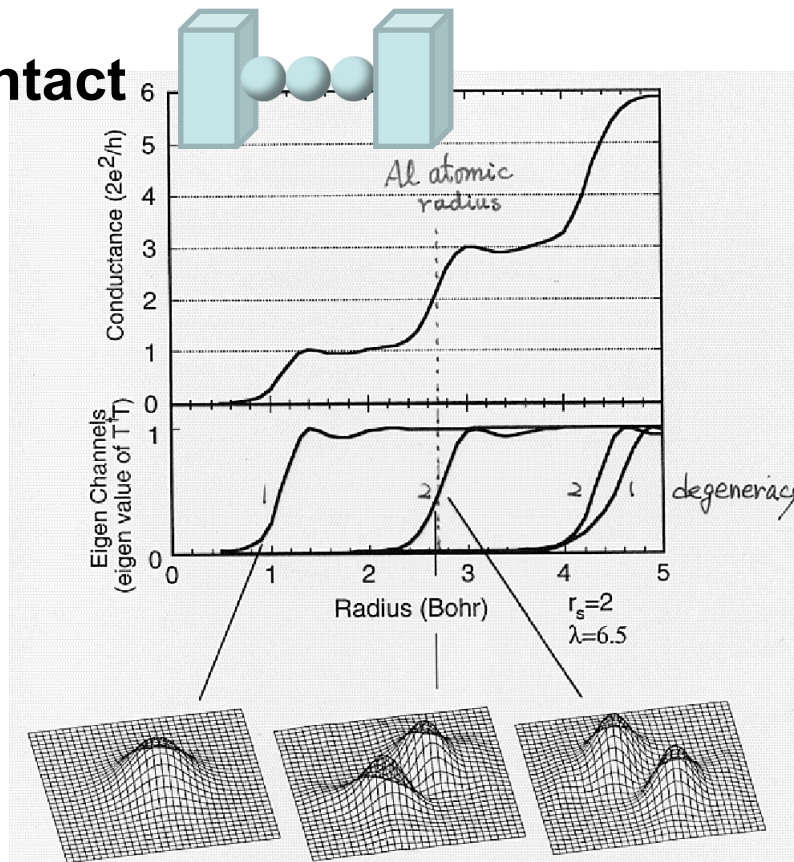
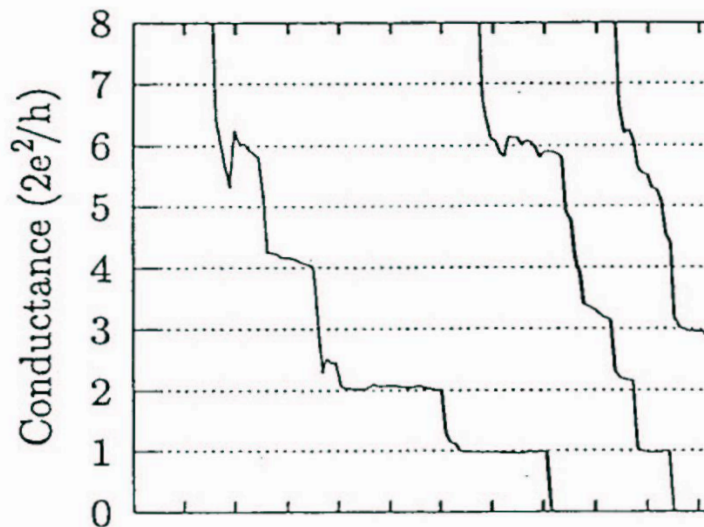


Conductance at the contact is close to the value of the quantization unit



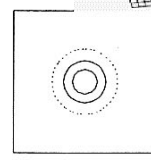
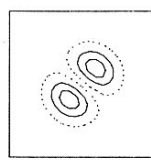
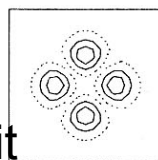
# Quantum conductance of Au point contact

By Prof. Lars Olesen



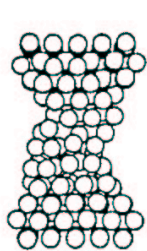
$$\frac{2e^2}{h} = \frac{1}{12.9k\Omega}$$

Quantization unit

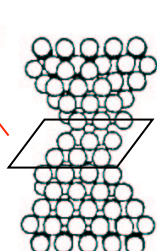


W tip wetted with Au

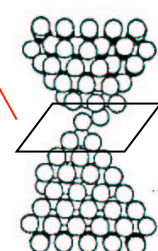
(a)



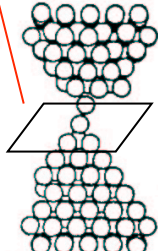
(b)



(c)



(d)



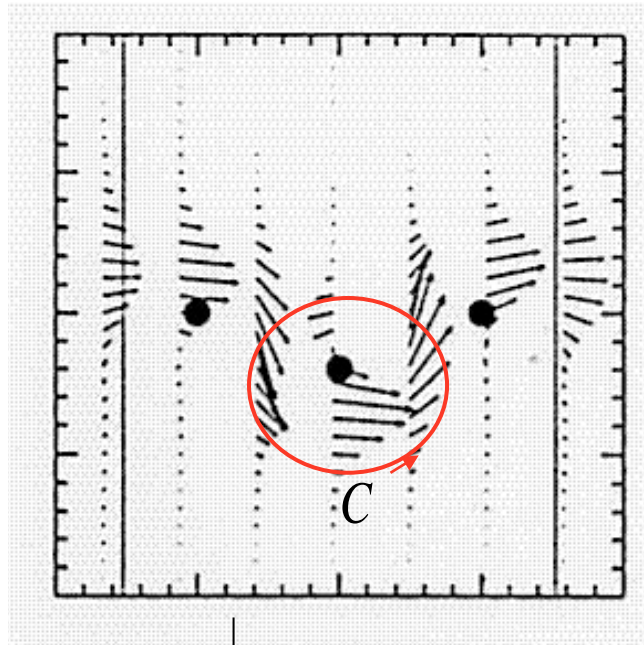
Au surface

Retracting the tip from the gold surface

**Electron behaves as a wave leading to quantum conductance**

Simulation by Prof.M.Brandbyge

# Loop current through bent Al atomic wire



$$\oint_{loop C} R\mathbf{j} \cdot d\mathbf{r} \neq 0$$

*loop C*



Contradiction with  
Ohm's law!

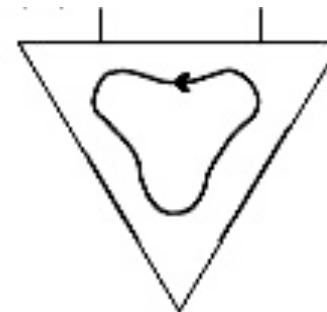
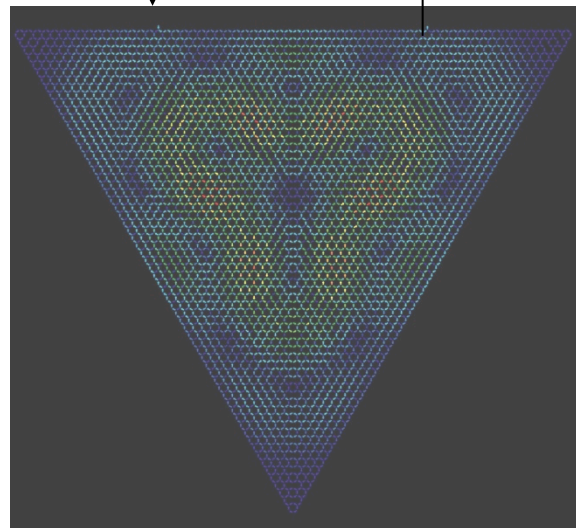


**Without voltage drop, i.e.,  
Without deriving electromotive  
Force, current flows.**



**Remarkable quantum  
effect**

# Loop current through triangle graphene





# From coherent to dissipating electron transport

$d(E - E\phi)$  for elastic tunneling

Electron transition rate

$$\bar{\Gamma}(V) = \frac{1}{R} \int dE \int dE' f(E) \{1 - f(E' + eV)\} P(E - E')$$

$$P(\Delta E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dt \exp \left[ J(t) + \frac{i}{\hbar} \Delta E t \right]$$

energy loss spectral function

$$J(t) \equiv \langle (\varphi(t) - \varphi(0)) \varphi(0) \rangle$$

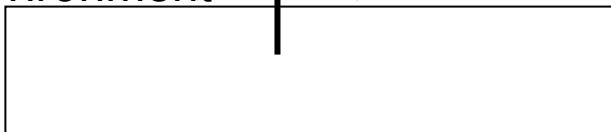
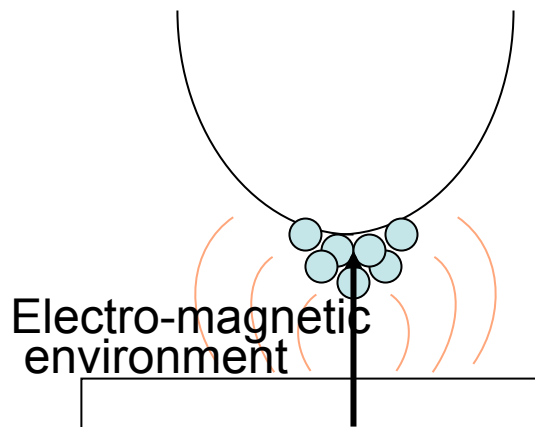
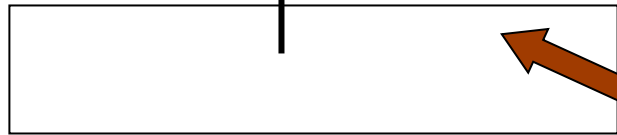
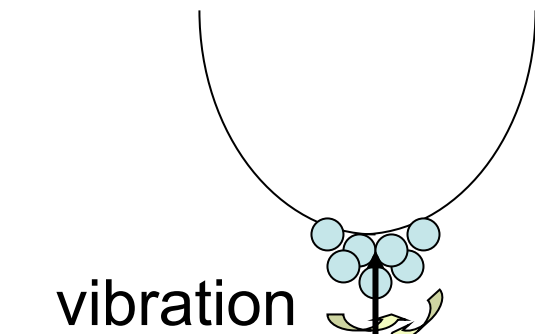
$$= \sum_{\lambda} \gamma_{\lambda}^2 \left\{ \coth \left( \frac{\beta \hbar \omega_{\lambda}}{2} \right) (\cos \omega_{\lambda} t - 1) - i \sin \omega_{\lambda} t \right\}$$

$$= \frac{2e^2}{h} \int_0^{\infty} \frac{d\omega}{\omega} \text{Re} Z_t(\omega) \left\{ \coth \left( \frac{\beta \hbar \omega}{2} \right) (\cos \omega t - 1) - i \sin \omega t \right\}$$

Electron-Vibration Coupling strength

$$\gamma_{\lambda} = \frac{\eta_{\lambda}}{\hbar \omega_{\lambda}} = \sqrt{\frac{\pi e^2 \alpha_{\lambda}}{h C \omega_{\lambda}^3}}$$

Normal mode intensity of electro-magnetic environment



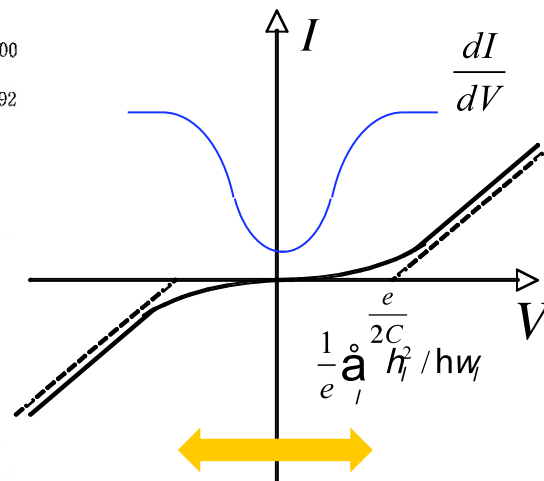
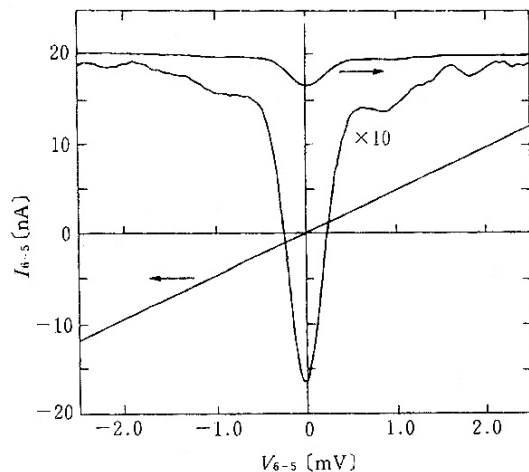
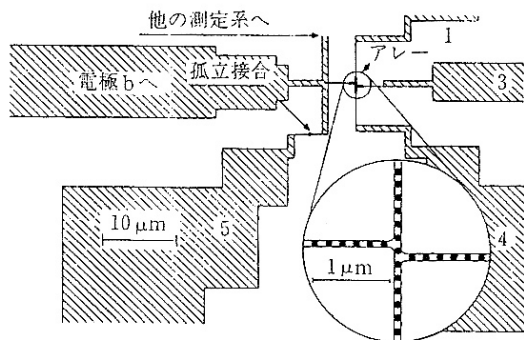
Inelastic tunneling spectroscopy

$$\frac{d^2 I}{dV^2} = \frac{e}{R} P(eV)$$

$$\frac{dI}{dV} = \frac{1}{R} \int_0^{eV} P(\Delta E) d\Delta E$$

Zero bias anomaly

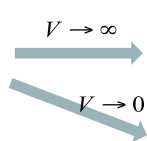
# The integrated energy loss spectral function and zero bias anomaly



Small gap of the conductance is opened between the unoccupied and the occupied sites; this is zero bias anomaly

$$\frac{d^2 I}{dV^2} = \frac{e}{R} P(eV)$$

$$\frac{dI}{dV} = \frac{1}{R} \int_0^{eV} P(E) dE$$



$$\begin{aligned} \xrightarrow{V \rightarrow \infty} \quad \frac{dI}{dV} &= \frac{1}{R} \\ \xrightarrow{V \rightarrow 0} \quad \frac{dI}{dV} &= 0 \end{aligned}$$

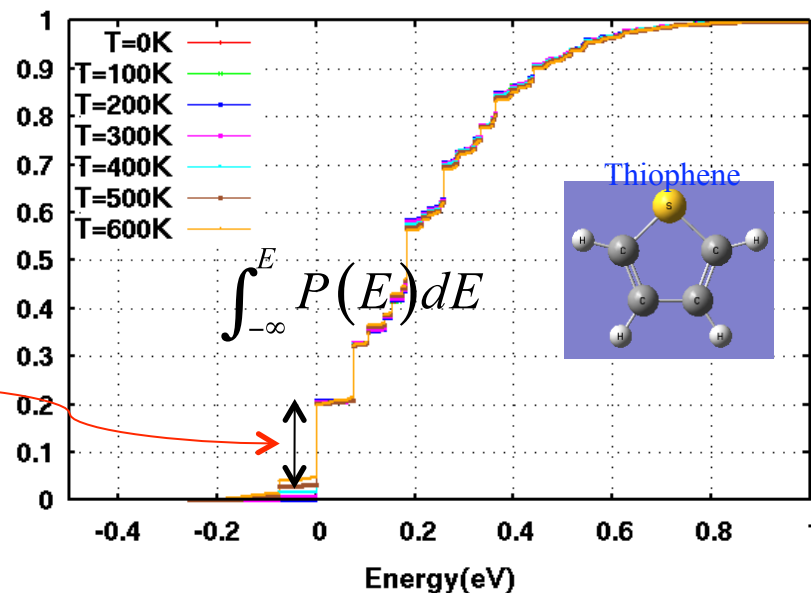
The step height at origin corresponds to the prob. of coherent process

$$G(0) = \exp \left[ -2 \sum_{\lambda} \gamma_{\lambda}^2 k_B T / \hbar \omega_{\lambda} \right] G^{(0)}$$



Branching ratio to the coherent process

Integral of Inelastic Spectrum Function



# Theory of dynamic AFM

# Dynamics of cantilever as an elastic body

What does the conventional harmonic oscillator model imply?

Equation of motion for **a harmonic oscillator**

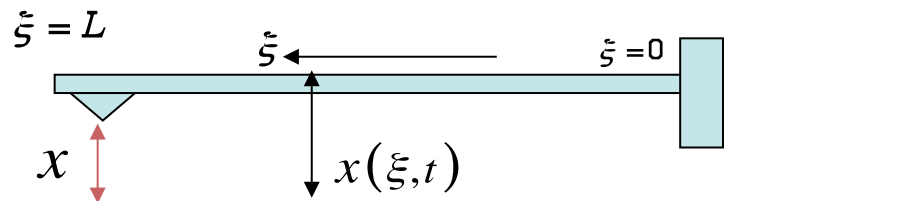
$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_c^2 x = F_{driv}(t) + F_{TS}(x)$$

Friction Coefficient   
 Cantilever Resonant Freq.   
 Deriving force Of cantilever   
 Tip-Surface Interaction force

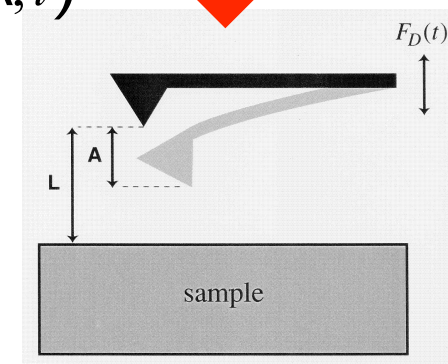
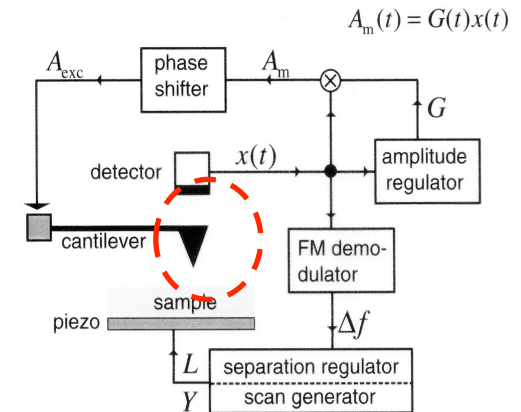


Equation of motion for **continuum elastic body**

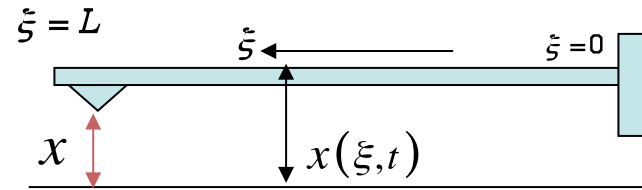
$$EI \frac{\partial^4 x(x,t)}{\partial x^4} + g \frac{\partial x(x,t)}{\partial t} + r \frac{\partial^2 x(x,t)}{\partial t^2} = F_{driv}(x,t) + F_{TS}(x,t)$$



The meaning of the parameters??



## Projection onto a normal mode



$$EI \frac{\partial^4 x(x,t)}{\partial x^4} + g \frac{\partial x(x,t)}{\partial t} + r \frac{\partial^2 x(x,t)}{\partial t^2} = F_{driv}(x,t) + F_{TS}(x,t)$$

$$x(\xi, t) = \sum_n x_n(t) \phi_n(\xi)$$

Eigen functions (Normal modes waves) obtained by

The coefficients satisfy the equation of motion of harmonic oscillator !!

$$\frac{d^2}{dt^2} x_n(t) + g \frac{d}{dt} x_n(t) + \omega_n^2 x_n(t)$$

$$= F_{driv}(t) + F_{TS}(t)$$

$$F_{driv}(t) = \frac{\int_0^L \tilde{F}_{driv}(\xi, t) \phi_n(\xi) d\xi}{\rho S_n}$$

$$F_{TS}(t) = \frac{\int_0^L \tilde{F}_{TS}(\xi, t) \phi_n(\xi) d\xi}{\rho S_n}$$

$$S_n = \int_0^L |\phi_n|^2 d\xi$$

$$EI \frac{d^4 \phi_n(\xi)}{d\xi^4} - \rho \omega_n^2 \phi_n(\xi) = 0 \quad \text{with} \quad \omega_n = \frac{C_n^2}{L^2} \sqrt{\frac{EI}{\rho}}$$

$$\phi_n(\xi) \Big|_{\xi=L} = \frac{d\phi_n(\xi)}{d\xi} \Big|_{\xi=L} = \frac{d^2\phi_n(\xi)}{d\xi^2} \Big|_{\xi=0} = \frac{d^3\phi_n(\xi)}{d\xi^3} \Big|_{\xi=0}$$

For uniform beam case

$$f_n(x) = \left( \cos(C_n x) - \cosh(C_n x) \right) + \frac{\cos(C_n L) + \cosh(C_n L)}{\sin(C_n L) + \sinh(C_n L)} - \left( \sin(C_n x) - \sinh(C_n x) \right)$$

Parameters are given by  $\cos(C_n L) \cosh(C_n L) + 1 = 0$

# Analysis of the forced harmonic oscillator model

## Standard Theory

amplitude

$$A = \frac{l}{2\sqrt{\left(\frac{f}{f_0} - 1 + r\right)^2 + h^2}}$$

frequency

Resonance curve

Frequency shift

$$\Delta f = rf_0 = -\frac{f_0}{2kA\pi} \int_0^{2\pi} F(A\cos\theta + L)\cos\theta d\theta$$

Tip-surface interaction force

Friction coefficient

Width of the resonant curve

$$h = \frac{1}{\pi\omega_0} \int_0^{2\pi} \gamma(A\cos\theta + L)\sin^2\theta d\theta$$

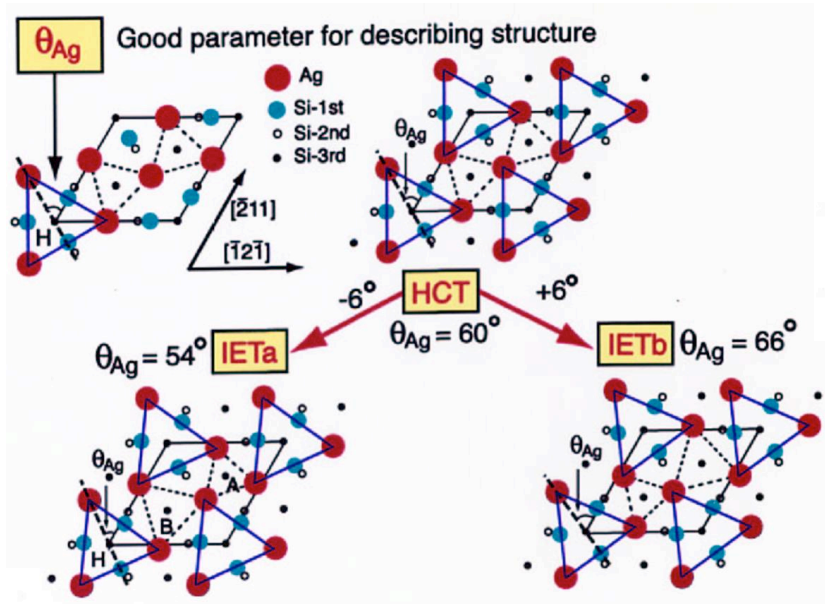
$$+ \frac{1}{2kA\pi} \int_0^{2\pi} F(A\cos\theta + L)\sin\theta d\theta$$

Hysteresis Force

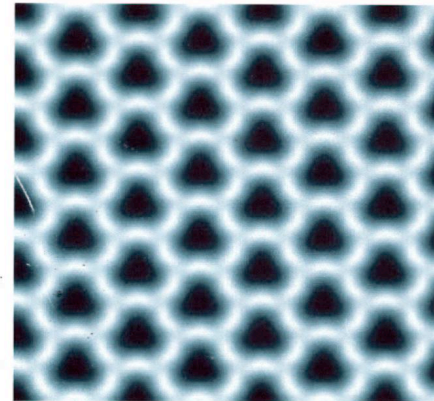
(First-Principles)  
Calculation  
with atomistic  
models

# Temperature dependence of NC-AFM image of $\text{Si}(111)\sqrt{3} \times \sqrt{3} - \text{Ag}$

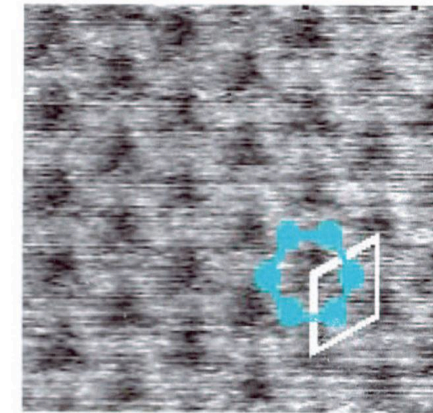
Model of fluctuating  
Surface structure



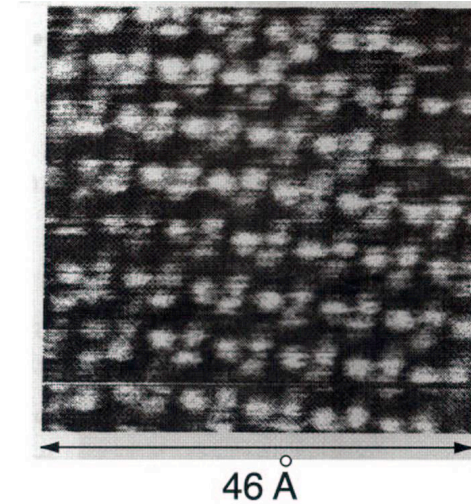
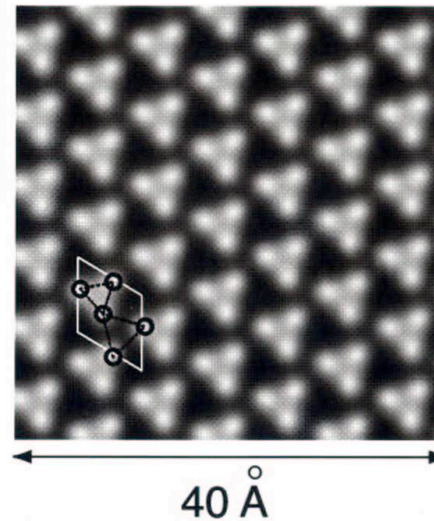
Theory  
 $d=4.50 \text{ \AA}$



Experiment  $T=300\text{K}$   
By Morita/Sugawara



$T=6.2\text{K}$



Remarkable temperature  
dependence is caused by the  
thermal adatom fluctuation  
Influenced by the tip

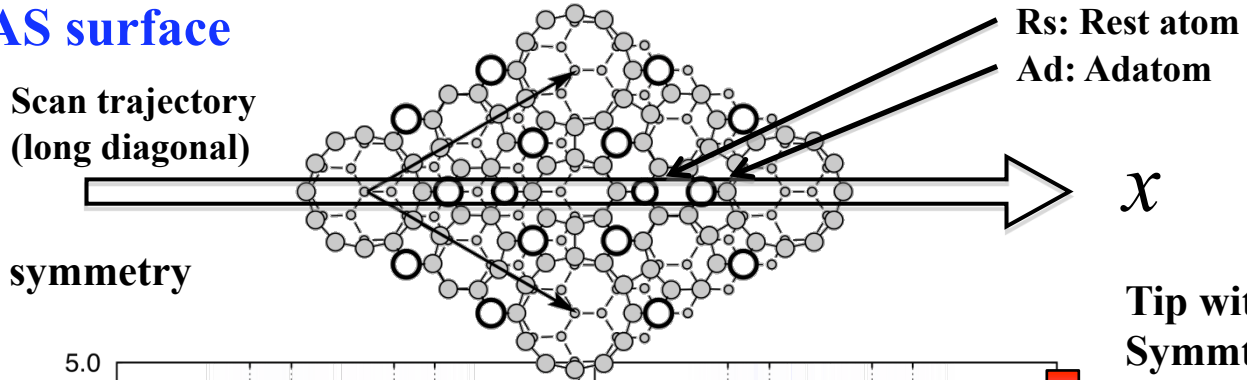
N.Sasaki, S.Watanabe and  
M.Tsukada, PRL 88(2002)046106

ncAFM

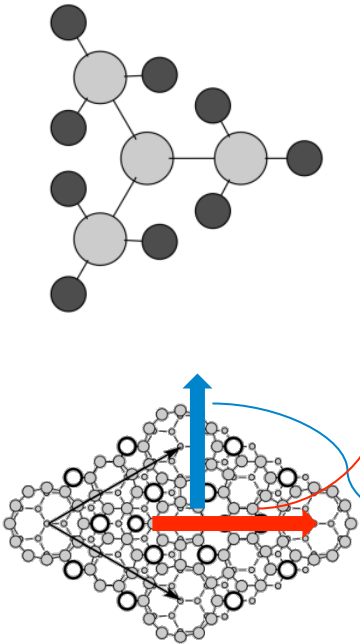
DFTB method

# Effect of tip structure on the 3D force map

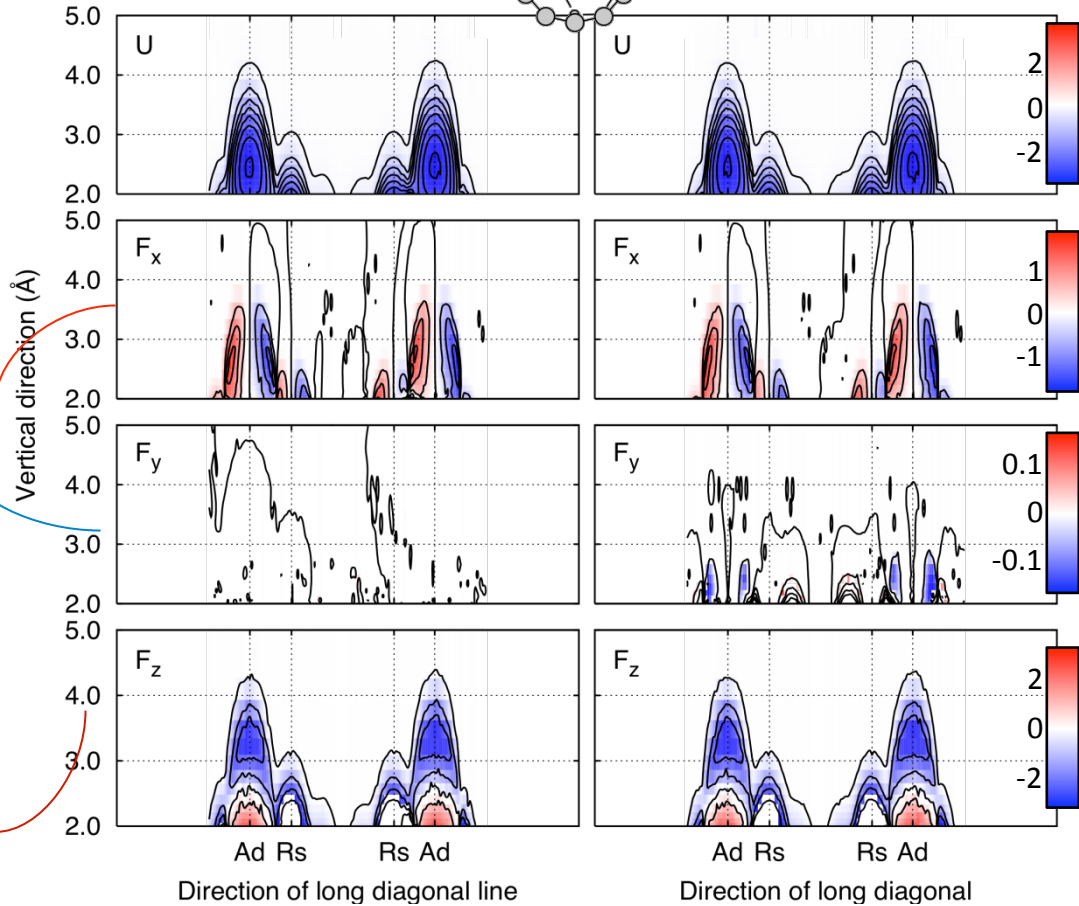
## Si(111)-(5x5)-DAS surface



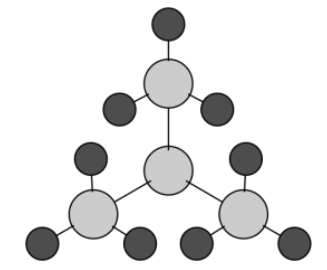
Tip with the mirror symmetry to x-axis



Vertical force To the surface



Tip without mirror Symmetry to x-axis



Ad: atom  
Rs: Rest atom



# Frequency shift image and dissipation image NaCl island on Cu(111)

Topography by  $\Delta v$

Dissipation image

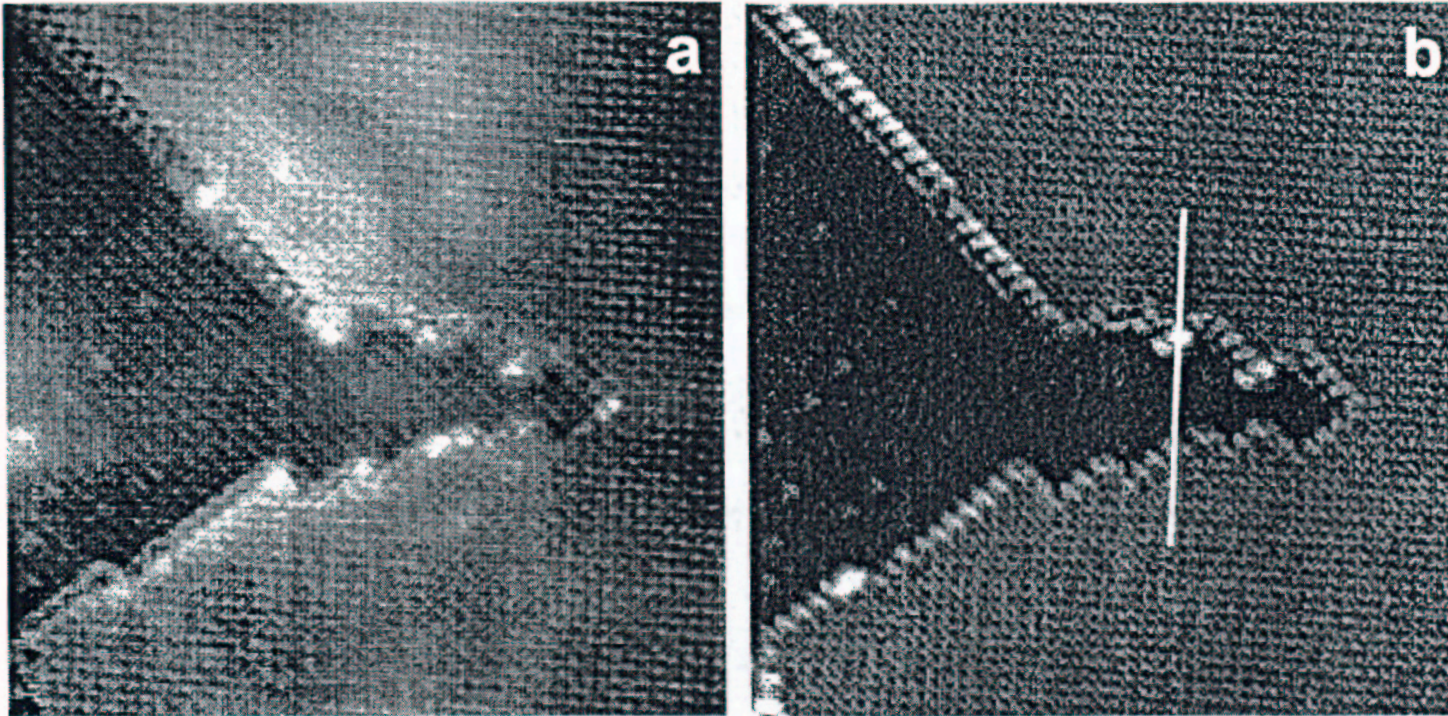


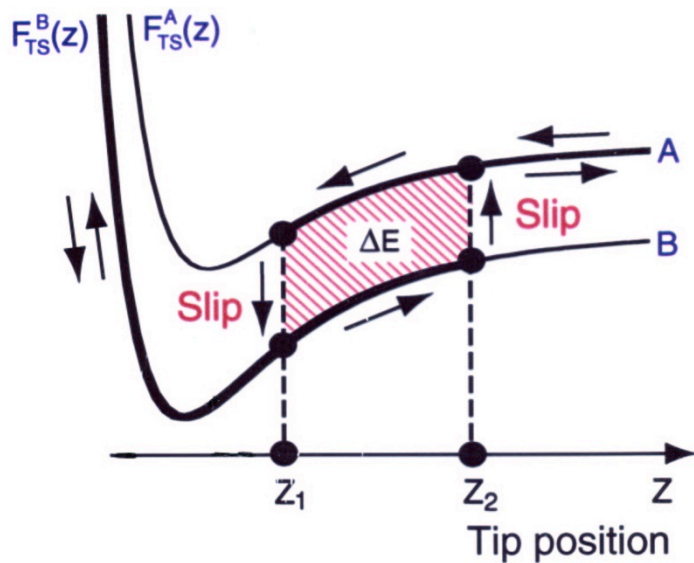
Figure 2: (a) Enlarged topography and (b)  $A_{ezc}$  images of the area mapped in Fig.1. Image size  $18 \times 18$  nm.

R.Bennewitz, A.S.Foster, L.N.Kantorovich, M.Bammerlin, Ch.Loppacher,  
S.Schar, M.Guggisberg, E.meyer and A.L.Shluger, Phys. Rev. B 62 (2000) 2074

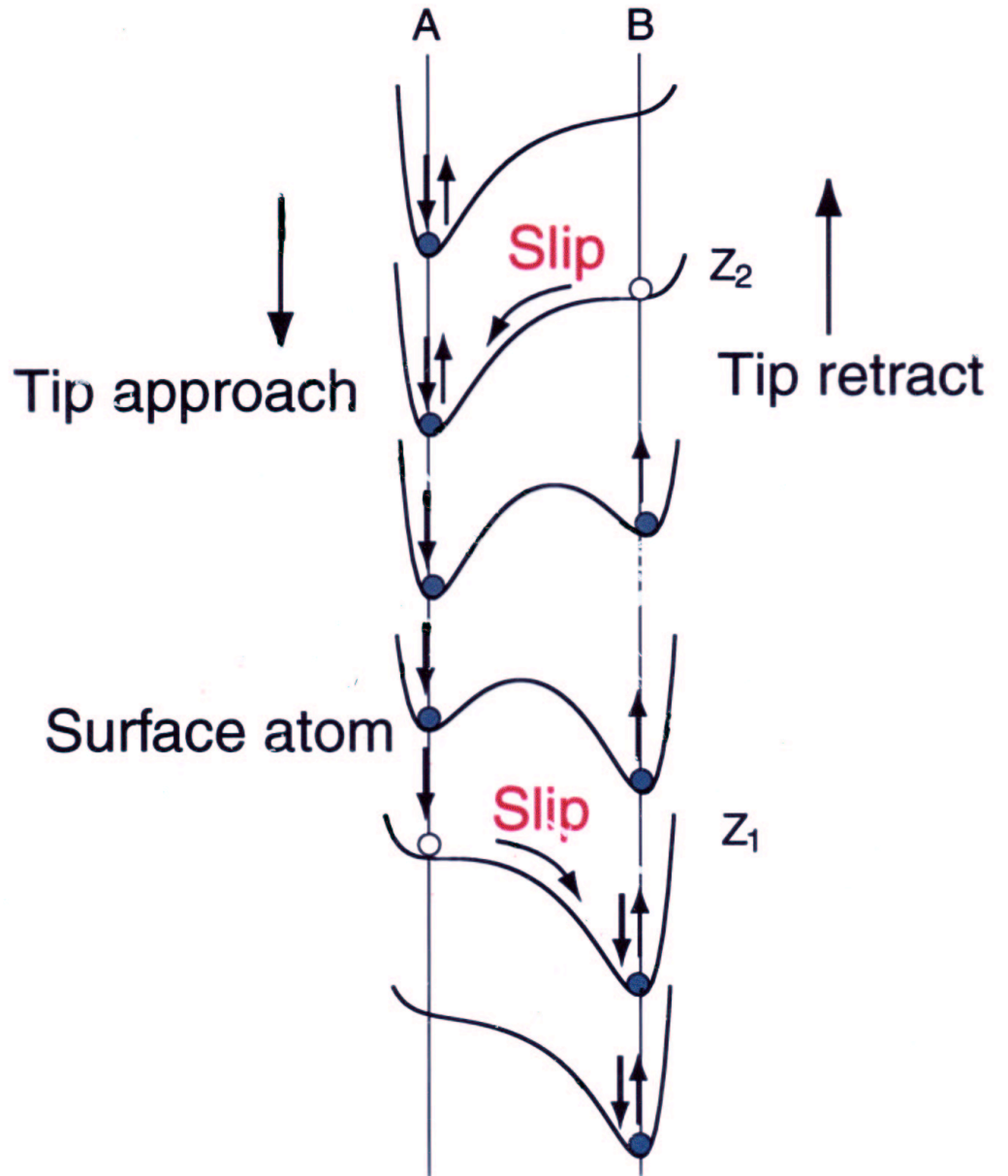
# Origin of dissipation

$$h = \frac{1}{\pi\omega_0} \int_0^{2\pi} \gamma (A \cos \theta + L) \sin^2 \theta d\theta$$

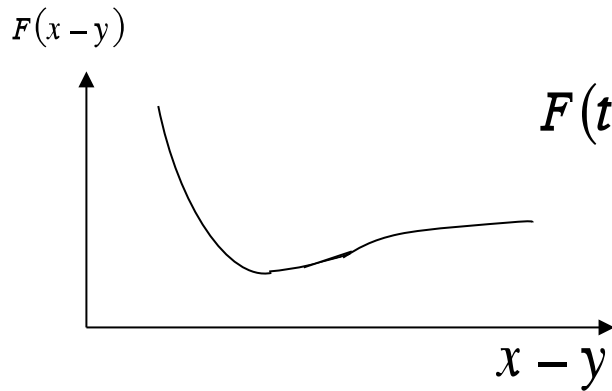
$$+ \frac{1}{2kA\pi} \int_0^{2\pi} F (A \cos \theta + L) \sin \theta d\theta$$



## Potential energy surface



# Dissipation force microscopy



Thermal Fluctuation

$$F(t) = \bar{x} - \bar{y} + \frac{\partial F}{\partial x} \delta x + \frac{\partial F}{\partial y} \delta y$$

Brownian motion of the tip/sample atoms

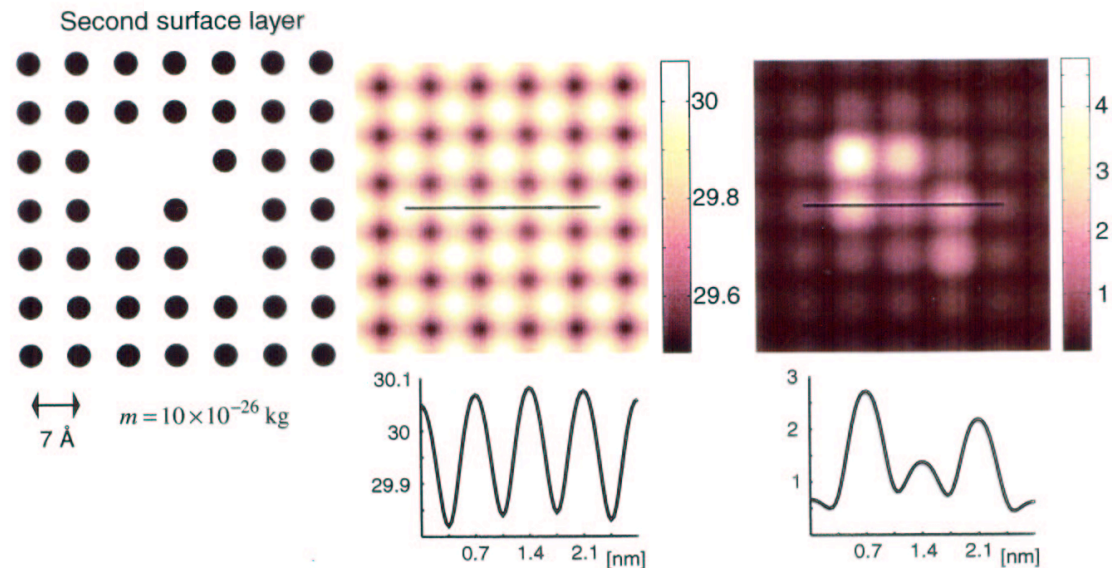
$$\langle \delta F(0) \delta F(t) \rangle = \left( \frac{\partial F}{\partial x} \right)^2 \langle \delta x(0) \delta x(t) \rangle + \left( \frac{\partial F}{\partial y} \right)^2 \langle \delta y(0) \delta y(t) \rangle$$

Fluctuation-Dissipation Theorem

$$\gamma = \frac{1}{Mk_B T} \int_0^{\infty} \langle \delta F(0) \delta F(t) \rangle dt$$

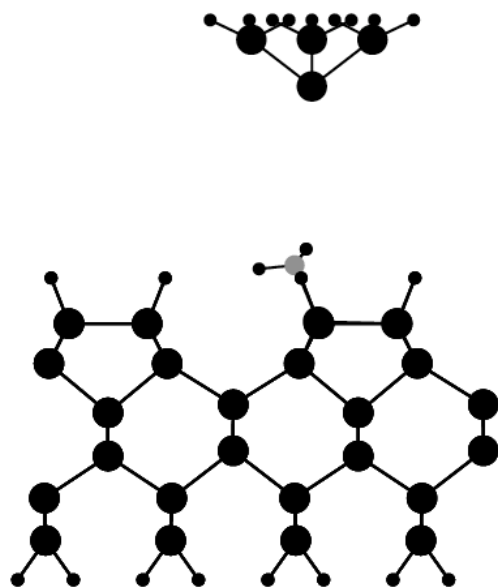
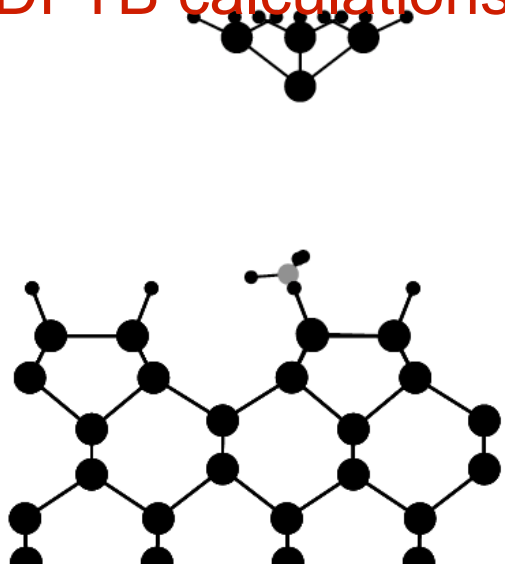
Detecting the stochastic motion of sample atoms with AFM?

M. Gauthier and M. Tsukada  
 Phys. Rev. Lett.  
 85(2000)5348  
 M. Gauthier and M. Tsukada  
 Surface Sci.  
 495(2001)204-210

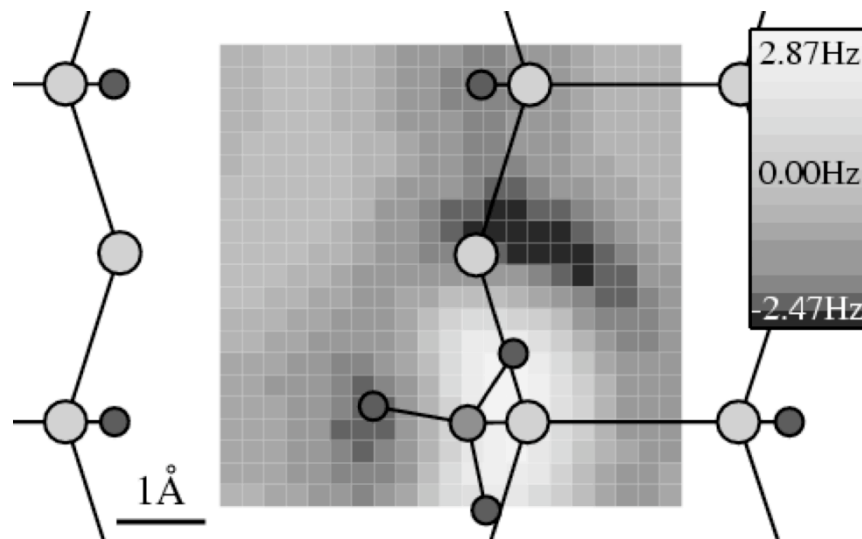


# Non-Contact AFM image of methyl group on Si(100)/H

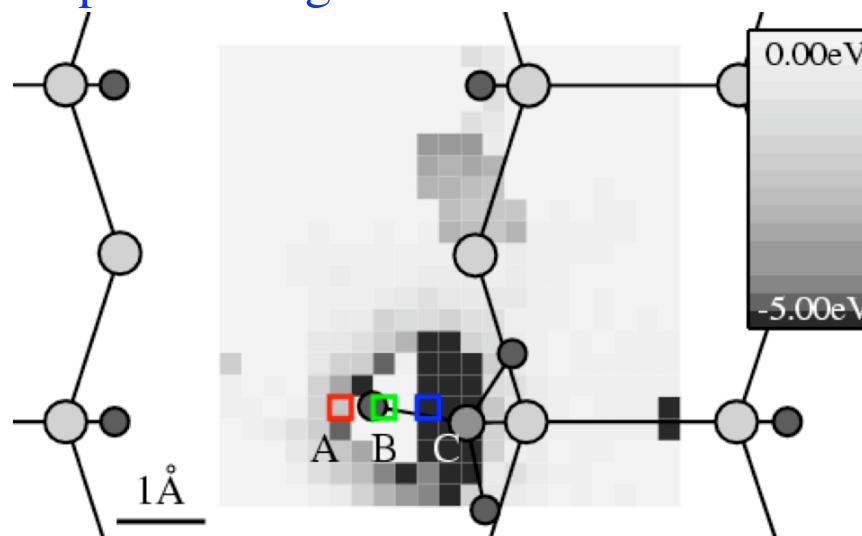
DFTB calculations



Frequency shift image Constant height

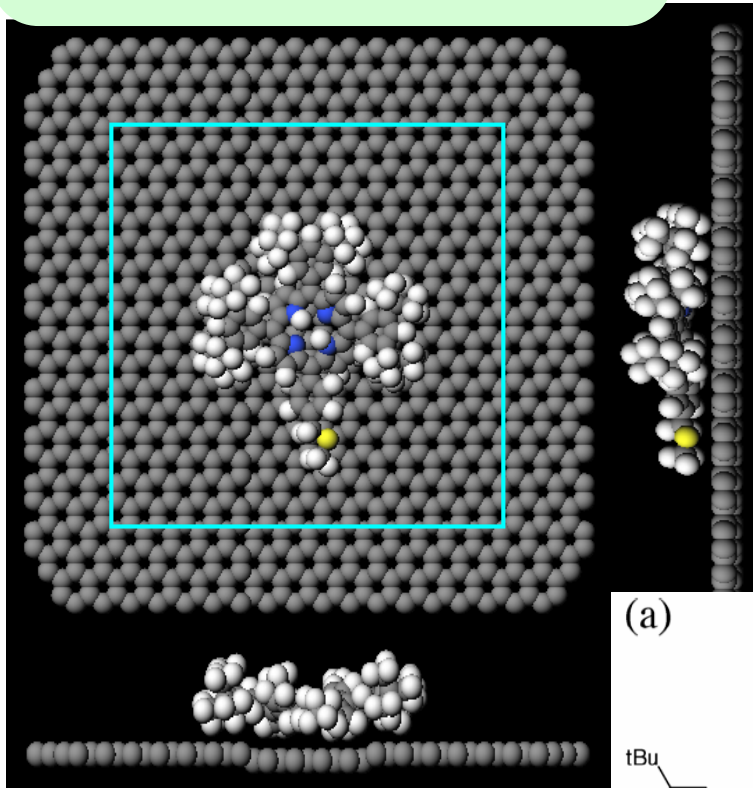


Dissipation image



## Molecular mechanics calculation

### AFM image of MSTBPP

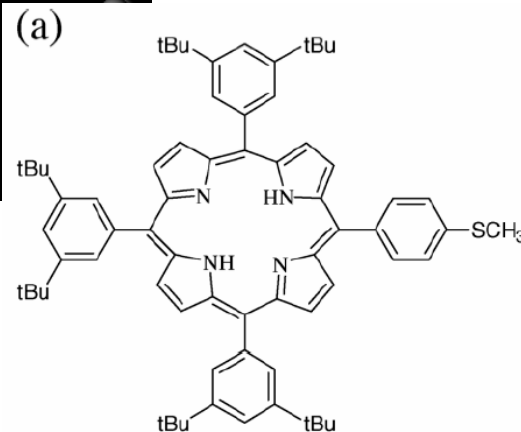


Depth = 0.5Å

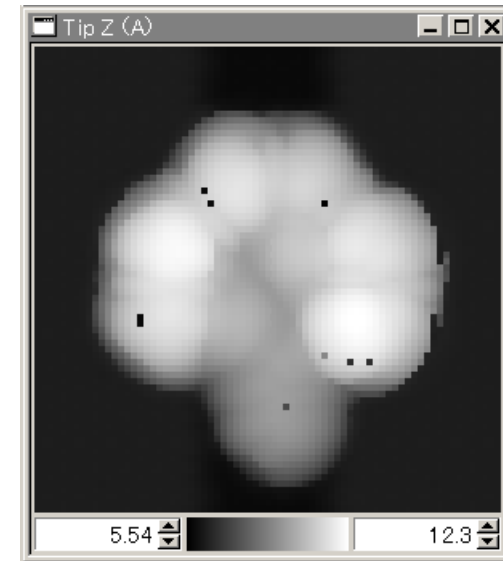
NC-AFM image of methylthiophenyl-tris-t-butylphenyl-porphyrin (MSTBPP) molecule observed by S.Tanaka' group

Hydrogen atom tip  
Fz = - 0.0005nN  
36Å×36Å (pixsize=0.5Å)

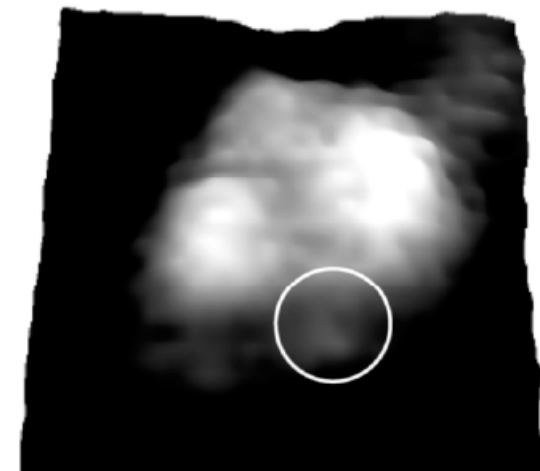
For fixed shape  
of molecule



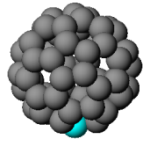
### Simulated constant force image



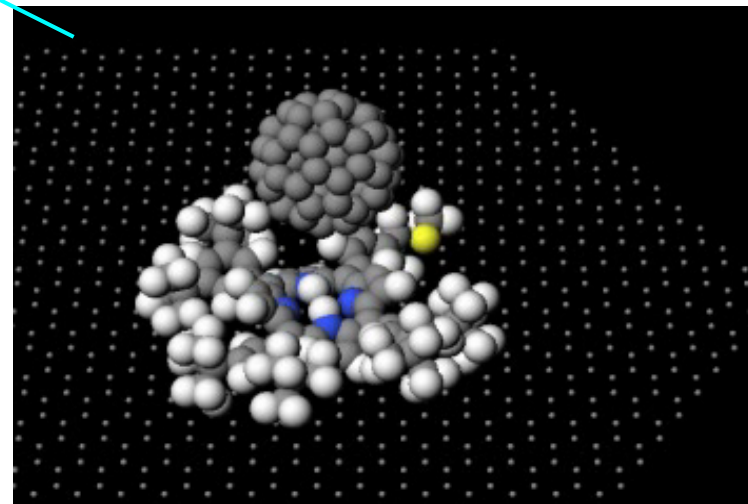
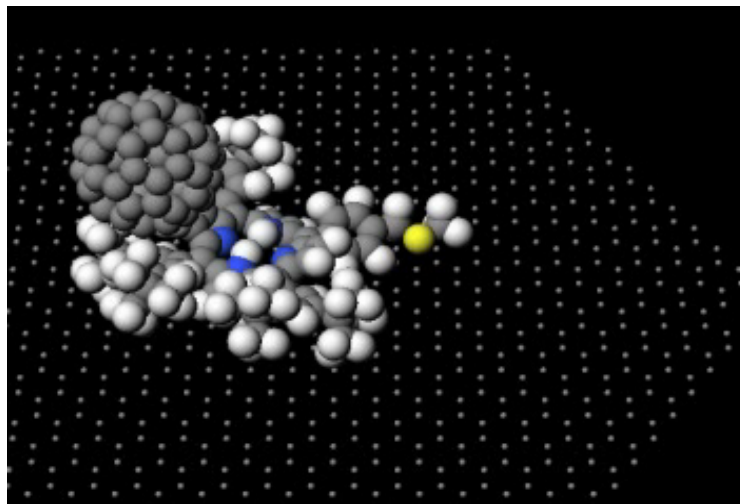
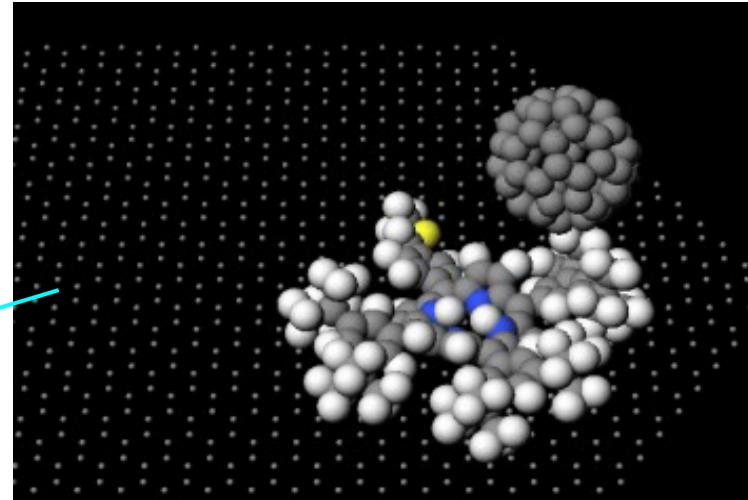
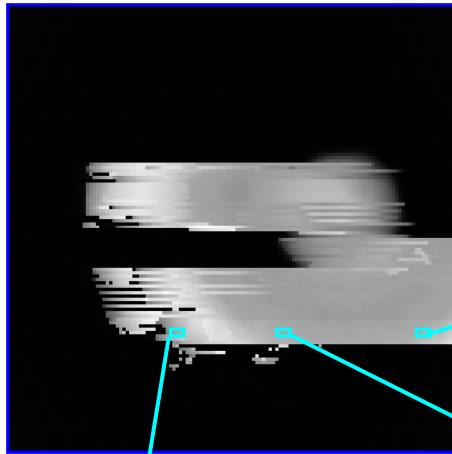
### Experimental image

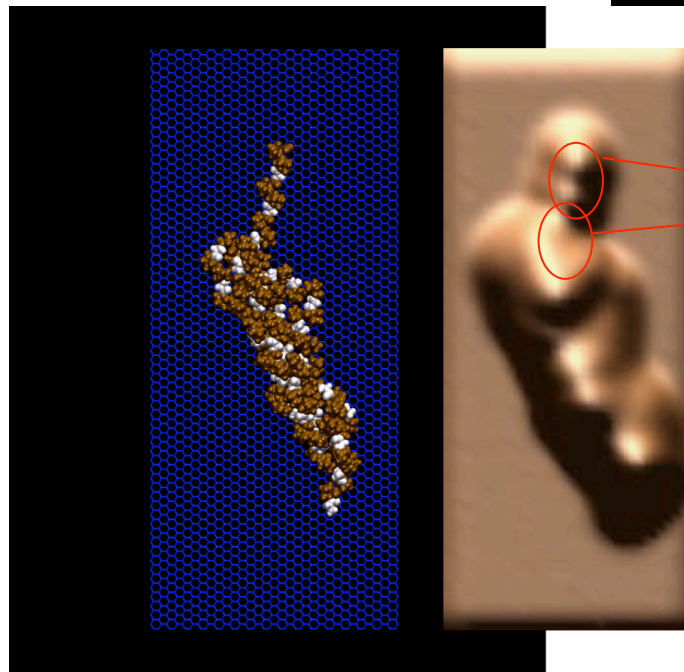
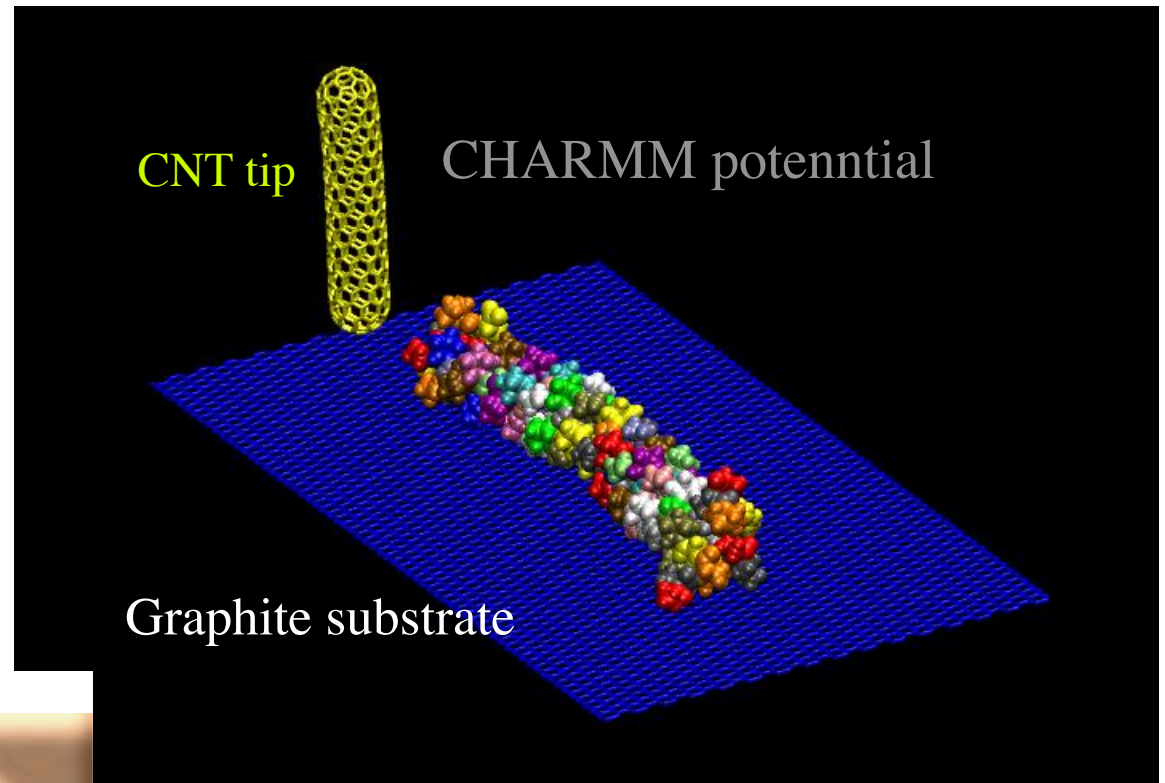
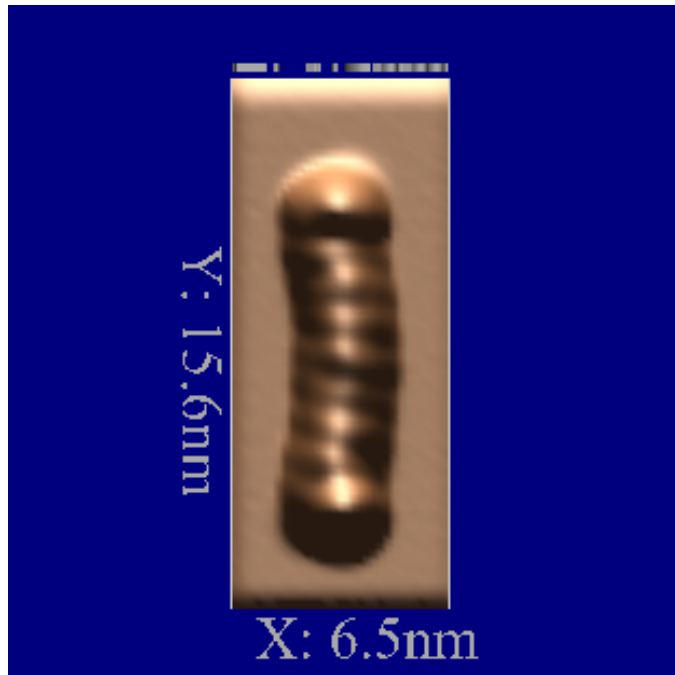


# NC-AFM image simulation with C60 tip for MSTBPP molecule, Constant frequency mode ~ Motion of the Molecule ~



Fullerene tip(2)  
 $\Delta f = + 2.0$  Hz





proline

**Constant mode AFM image  
simulation of a model collagen**

**Good correspondence  
with experiments!**

the tip recognizes  
height difference  
between PRO and GLY.

# A rapid simulation method of AFM images

Calculation by

Interaction  
force

Geometrical  
condition



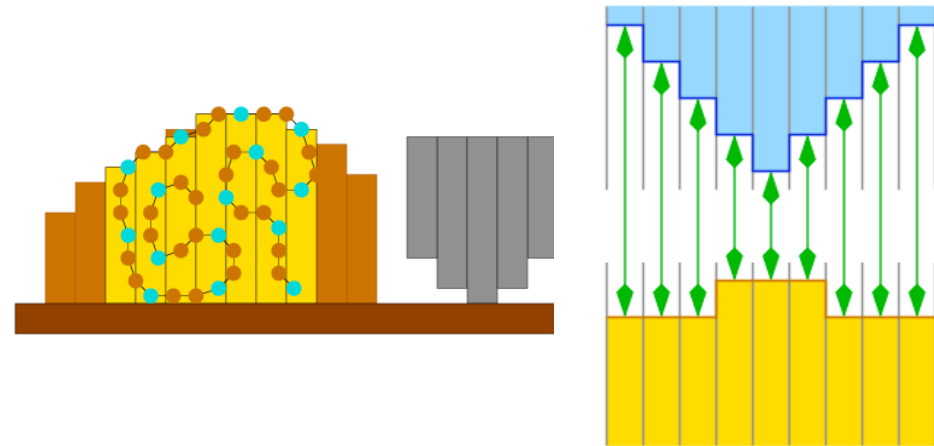
Computation time  
2 weeks with the  
usual WS



Computation time  
1 sec the usual PC

## Secret of rapid calculation

Using only geometrical  
condition

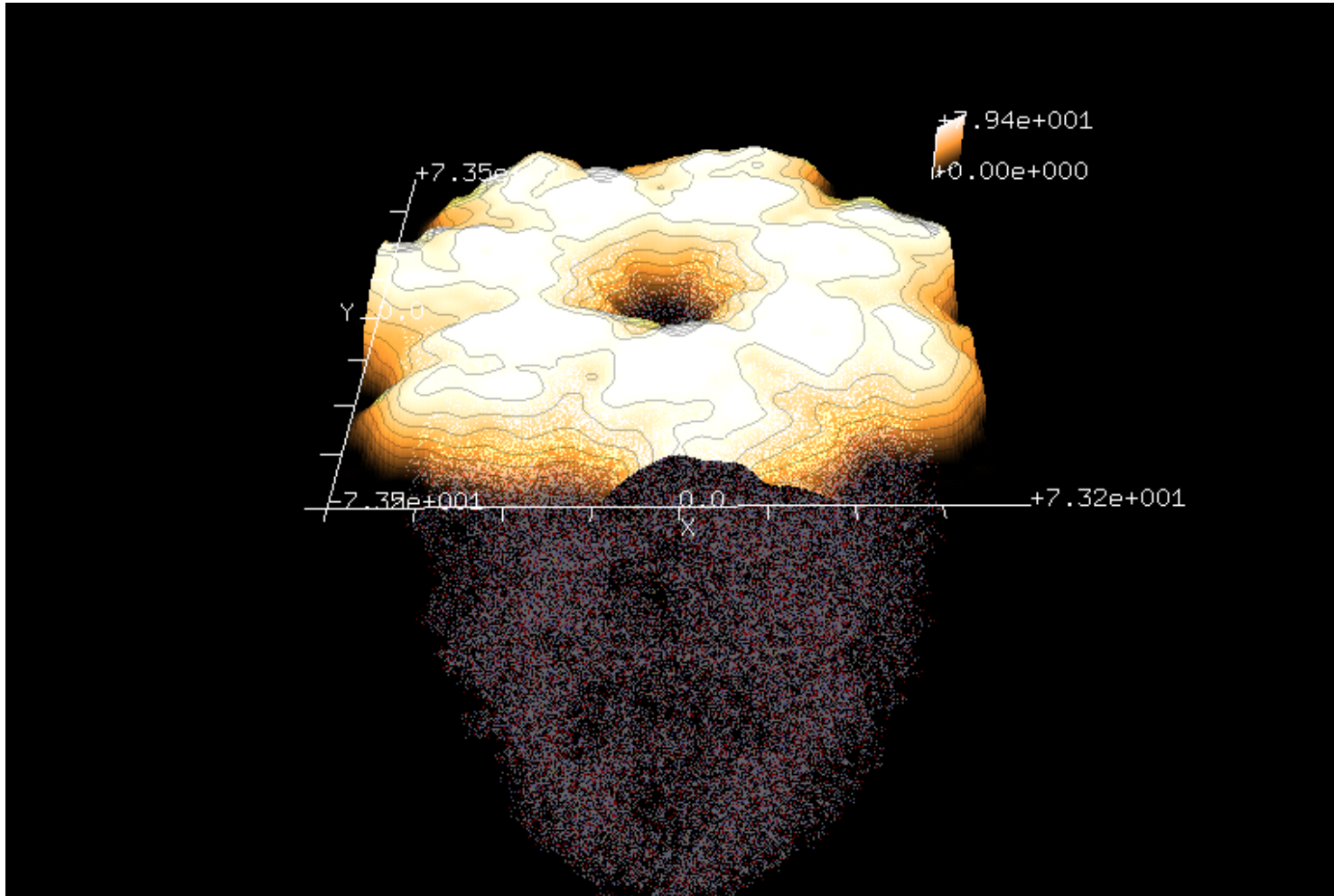


1. Divide each of the tip and the sample into fine meshes.
2. The highest atom in the mesh defines its height
3. Approach the tip vertically to the sample, until they touch each other.
4. The height of the tip at the touch defines the sample shape.



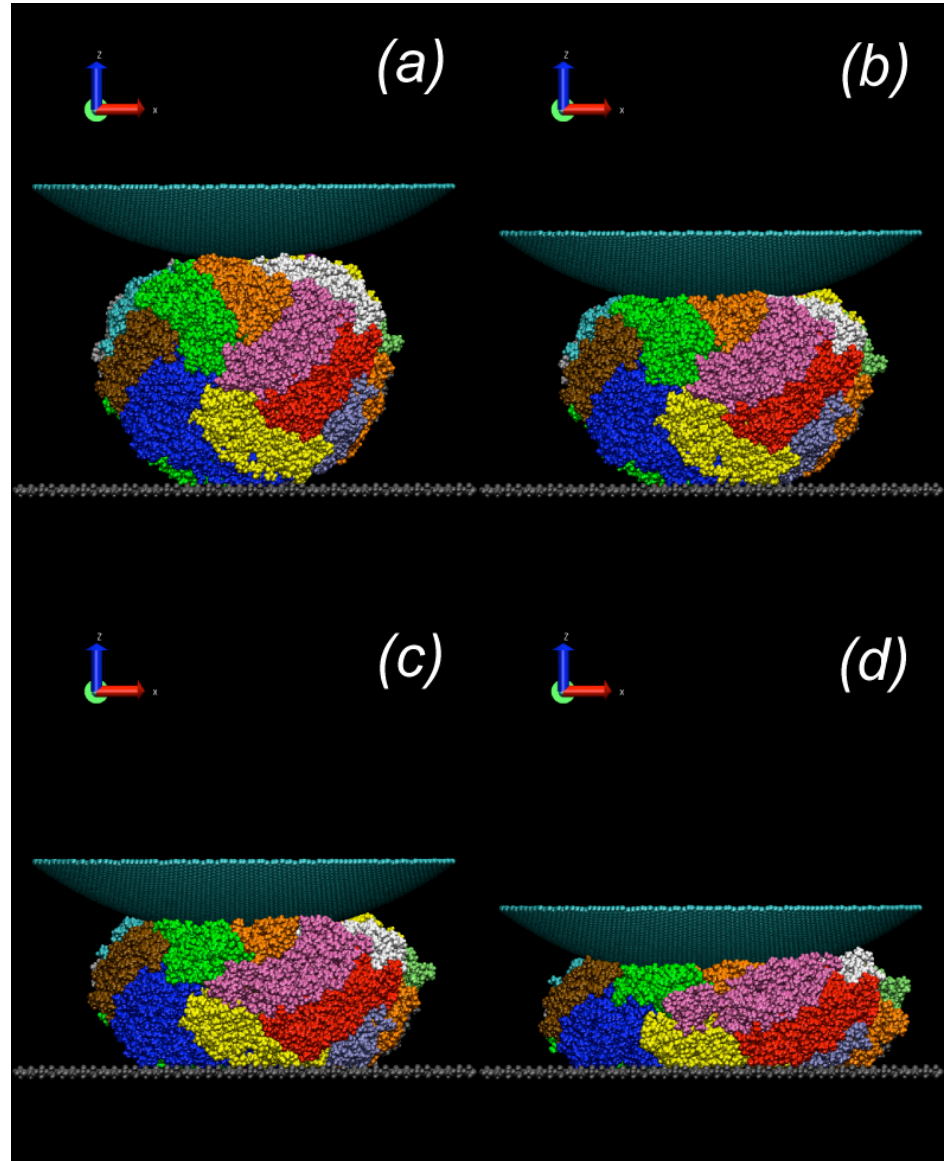
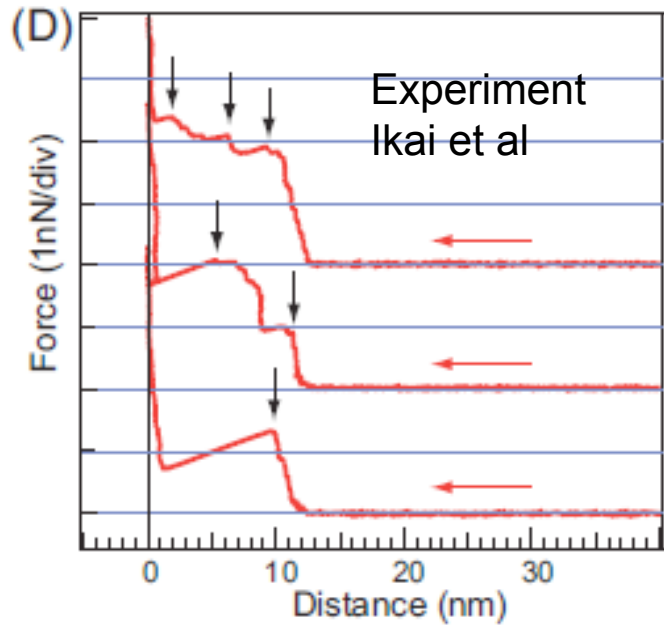
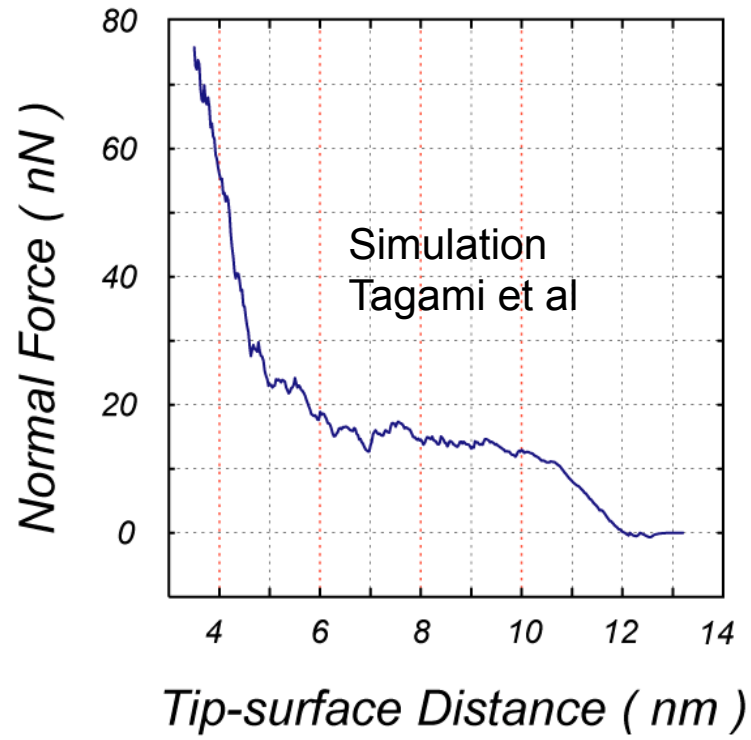
# Fast AFM image simulator for protein molecules

## The case of GroEL



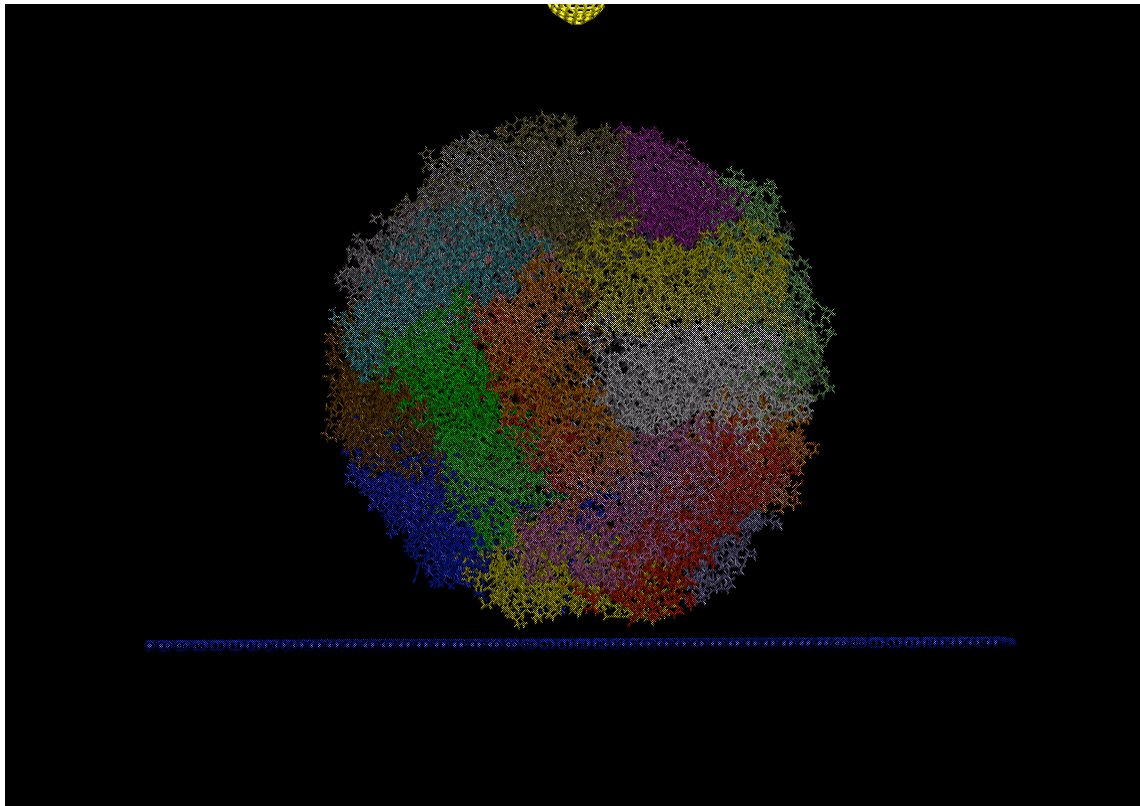
# Theory of nano mechanics of protein molecules

# Compression of apo-ferritin



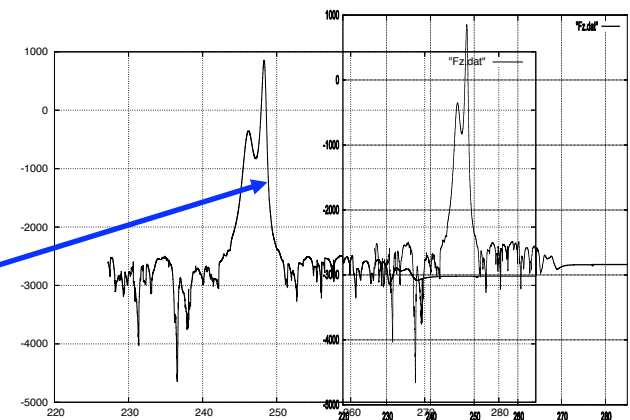
# Penetration of CNT tip through apo-ferritin

A simulation of nano-mechanical experiment on protein molecule



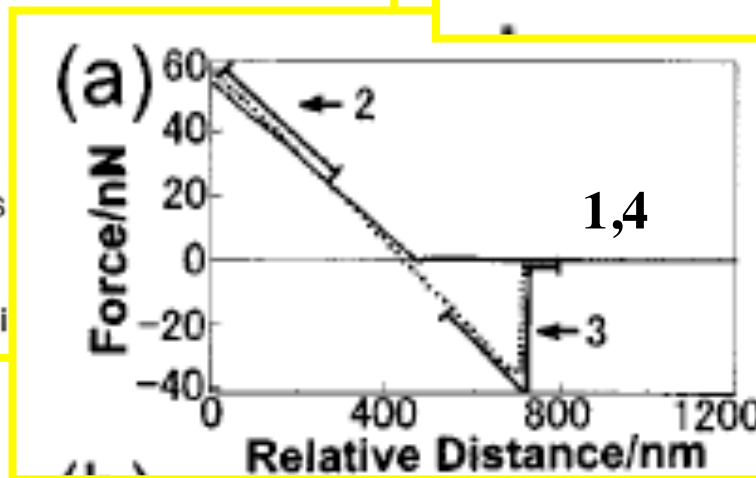
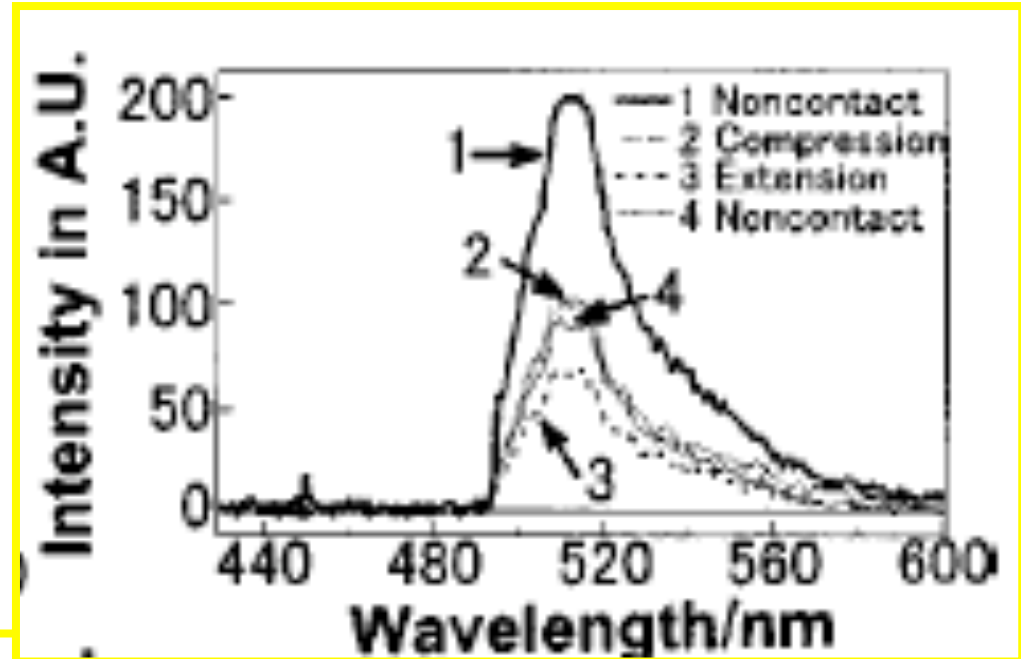
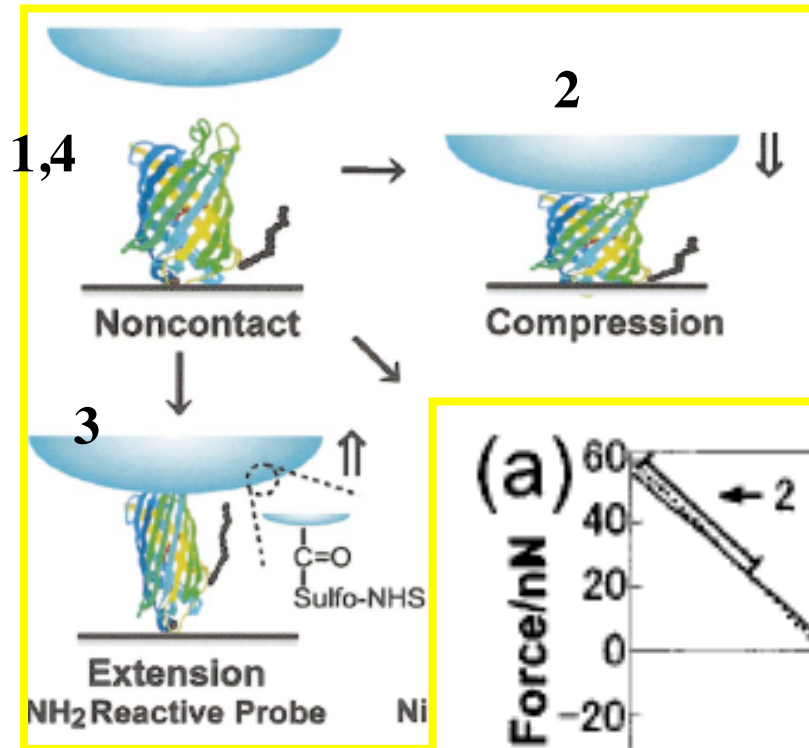
Tip velocity is 0.125Å/ps  
( Steered MD, T=0K )

Force is calculated by MD  
with Langevin method



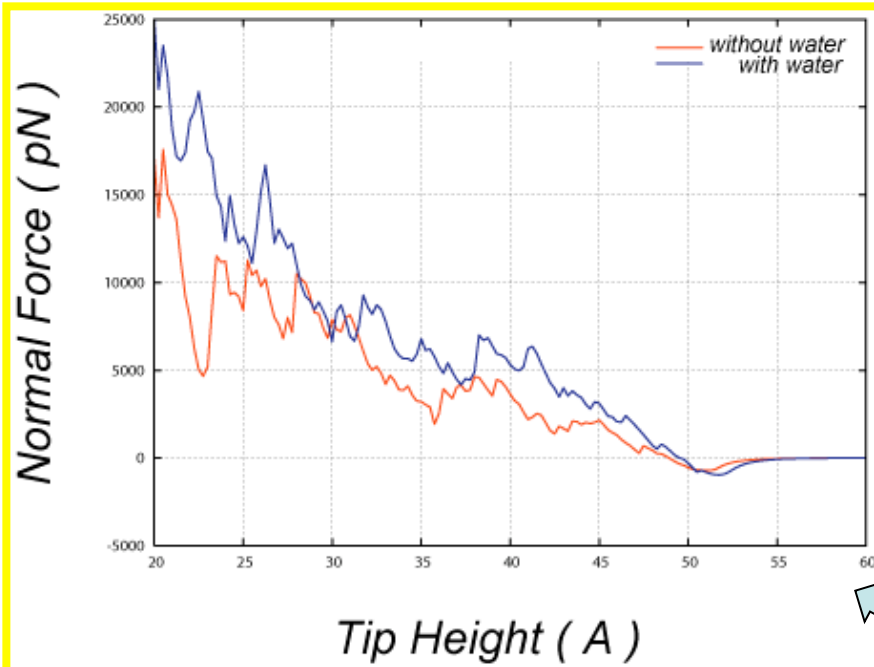
Large force at the instance  
of the penetration

# The effect of compression/elongations on the fluorescence of GFP

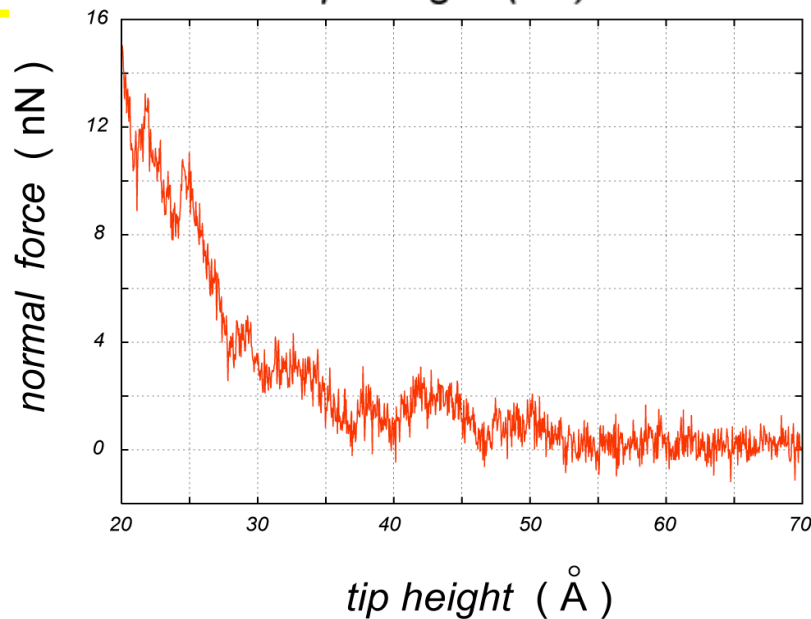
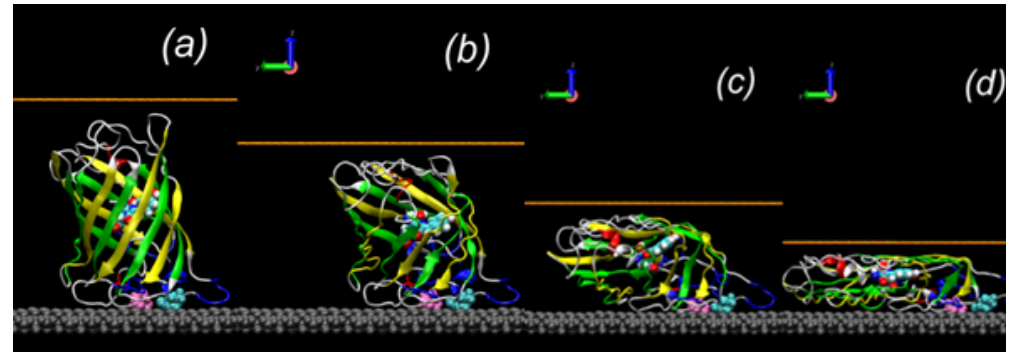


Remarkable quenching of the fluorescence was observed by Ikai et al

# Simulation of compression of GFP by a flat tip

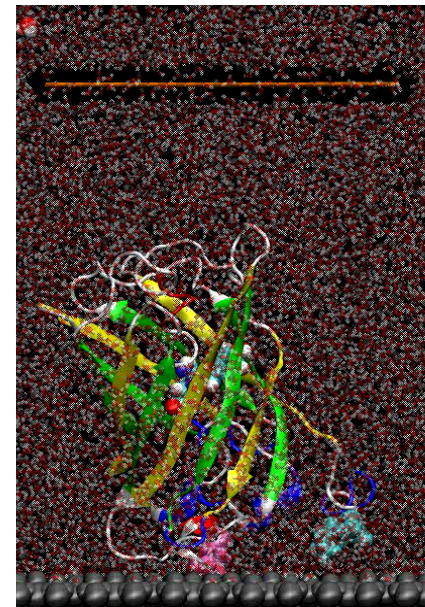


- Tip model ; graphite substrate : mica
- At each height, structure and the force are
- Calculated by CG and MD method



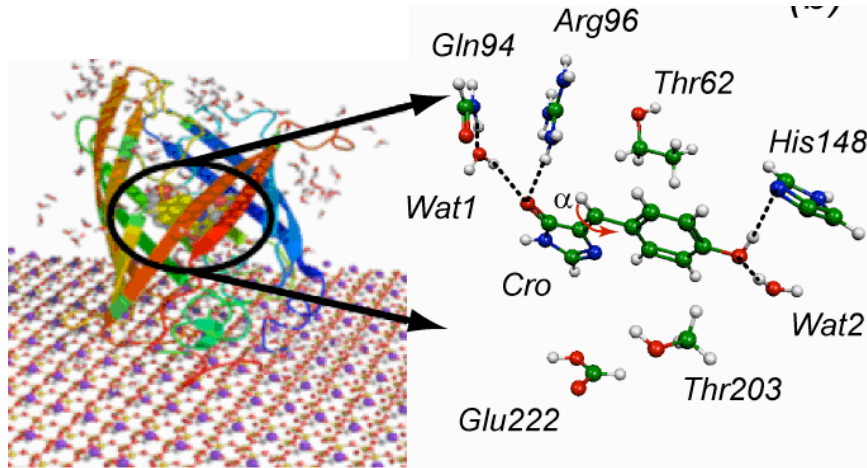
in vacuum

in water



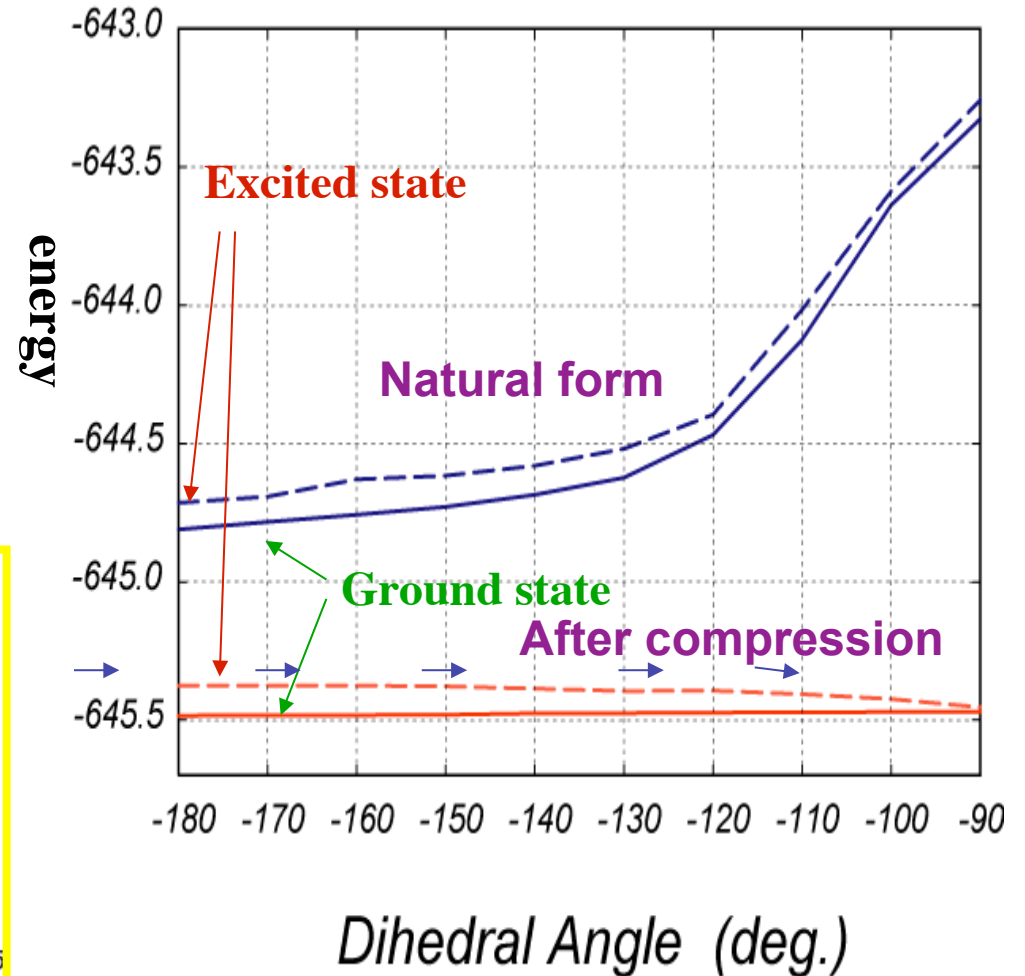
# What happens at Compression of GFP

Vertical excitation energy:  
 calculated 455nm  
 (observed:465nm)

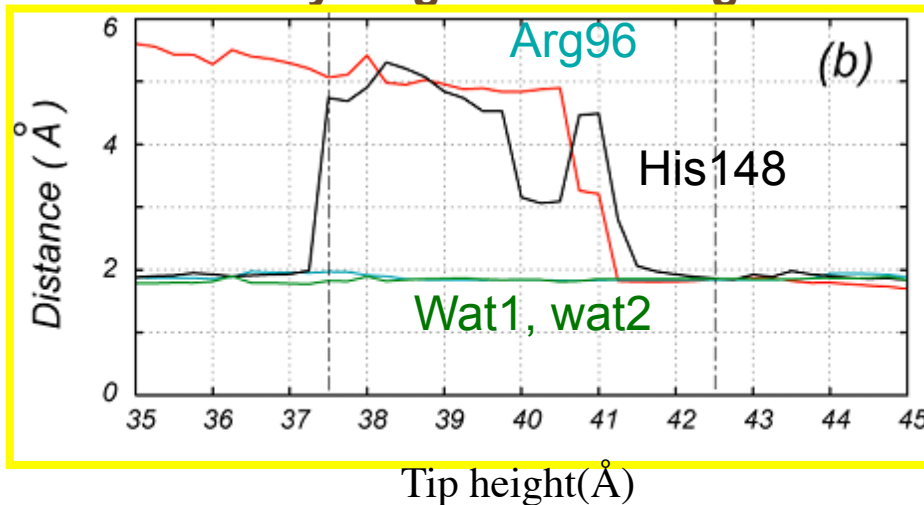


On the compression Barrier of the Rotation disappears

→ Non-radiative processes



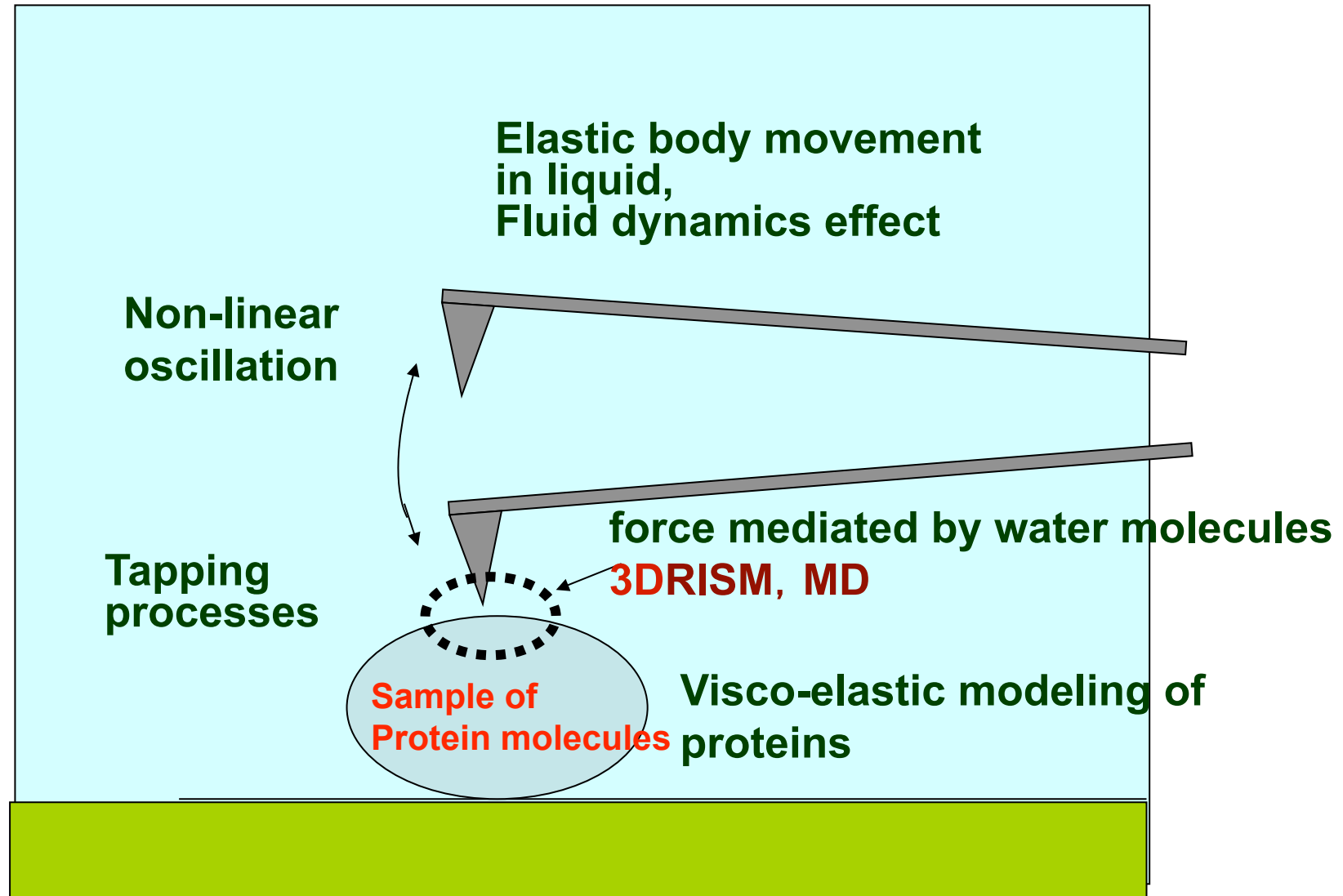
Hydrogen bond length



# Theory of dynamic AFM in liquids

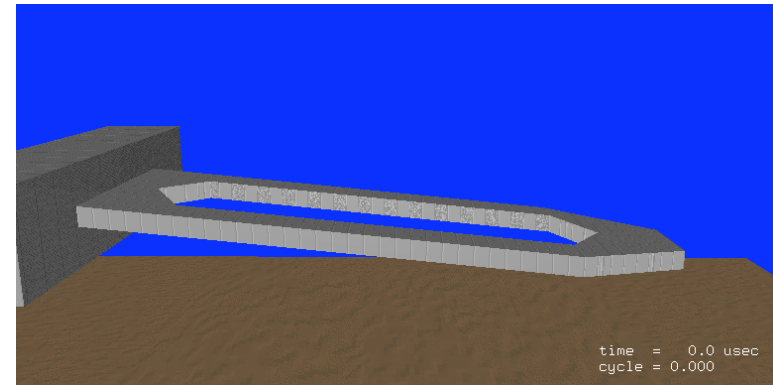


# Problems of the simulation methods for the tapping mode AFM in liquids

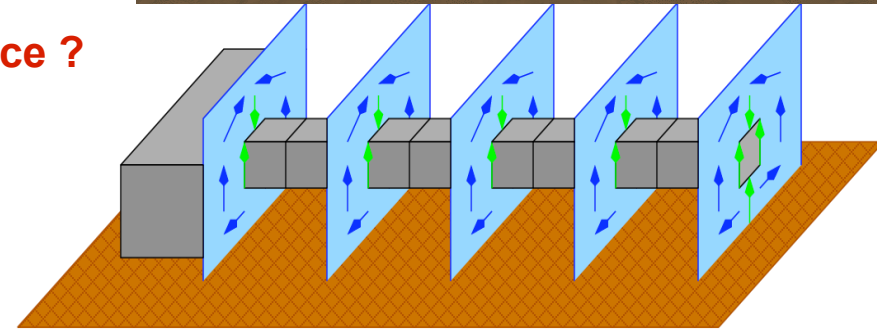


# A simplified model of 3D elastic-body/fluid combined system

- 1) Resonant Curve?
- 2) Nonlinear Effect?
- 3) Effect of the tip substrate distance ?



**Cantilever:**  
1D elastic beam



$$\rho S(z) \frac{\partial^2}{\partial t^2} h(z) = - \frac{\partial^2}{\partial z^2} EI(z) \frac{\partial^2}{\partial z^2} h(z) + F^{\text{liq}}(z)$$

**Water:**  
2D incompressible viscous fluid  
for each 2D cross section

$E$ ; Young's modulus  
 $I$ ; moment of the cross section

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \frac{1}{\text{Re}} \Delta \mathbf{v}$$

Re; Reynolds number

# Method for fluid dynamics on 2D

Flow function

$$\Psi \implies v_x = +\frac{\partial \psi}{\partial y} \quad v_y = -\frac{\partial \psi}{\partial x}$$

vorticity

$$\omega \implies \omega = \partial_x v_y - \partial_y v_x \quad \longrightarrow \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

From Navier-Stokes eq.  $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \frac{1}{\text{Re}} \Delta \mathbf{v}$

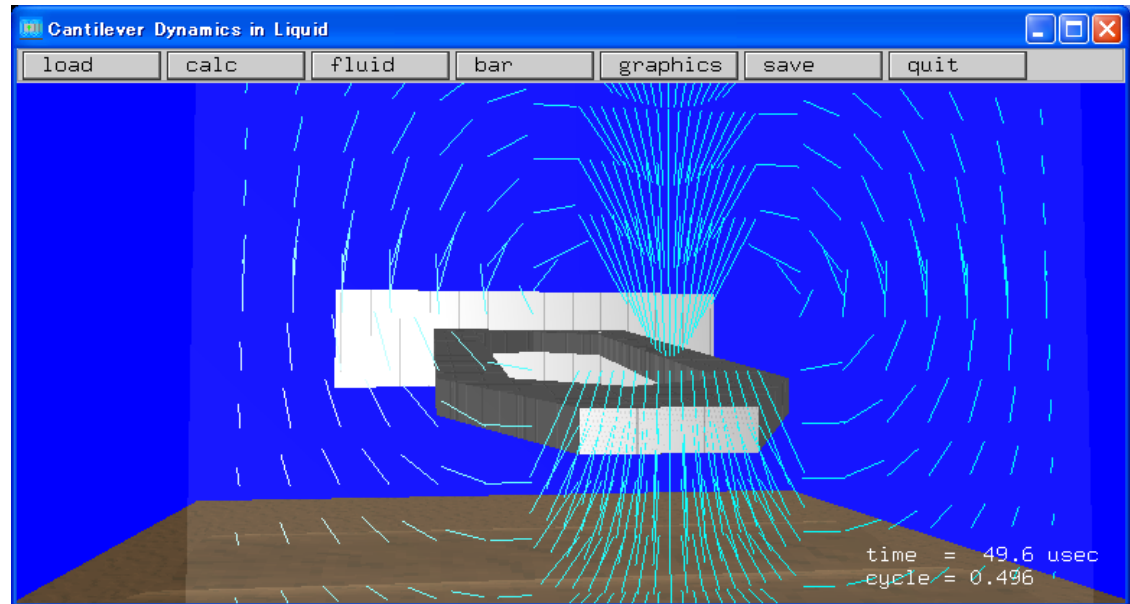
$$\frac{\partial \omega}{\partial t} = \left[ \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} \right] + \frac{1}{\text{Re}} \left[ \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right]$$

negligible

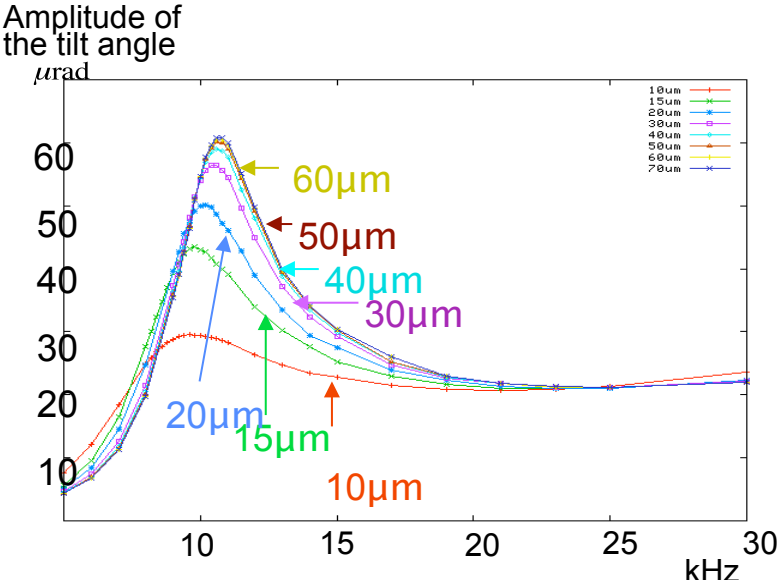
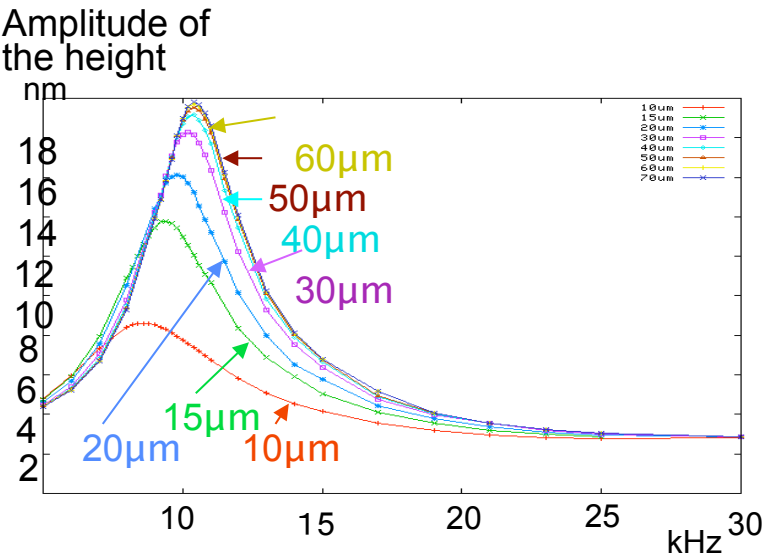
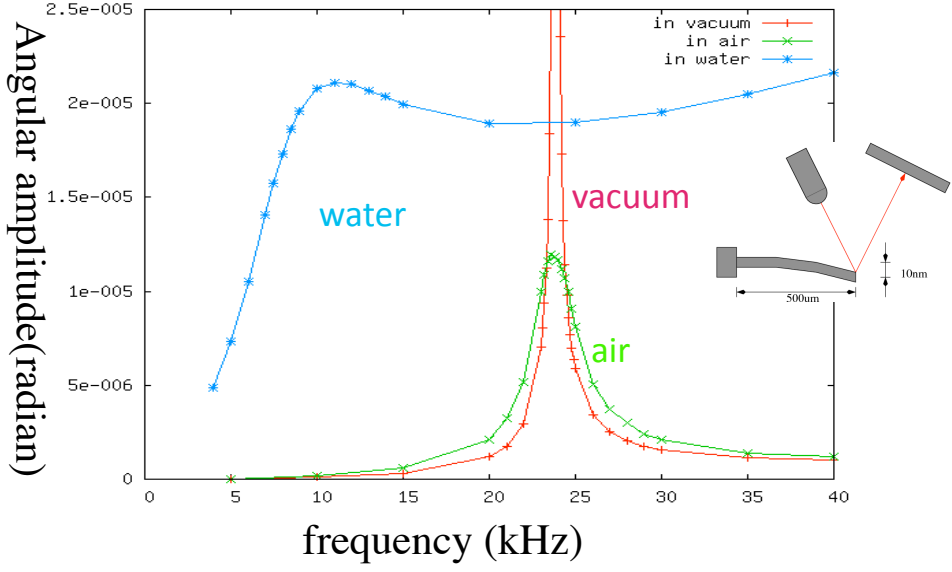
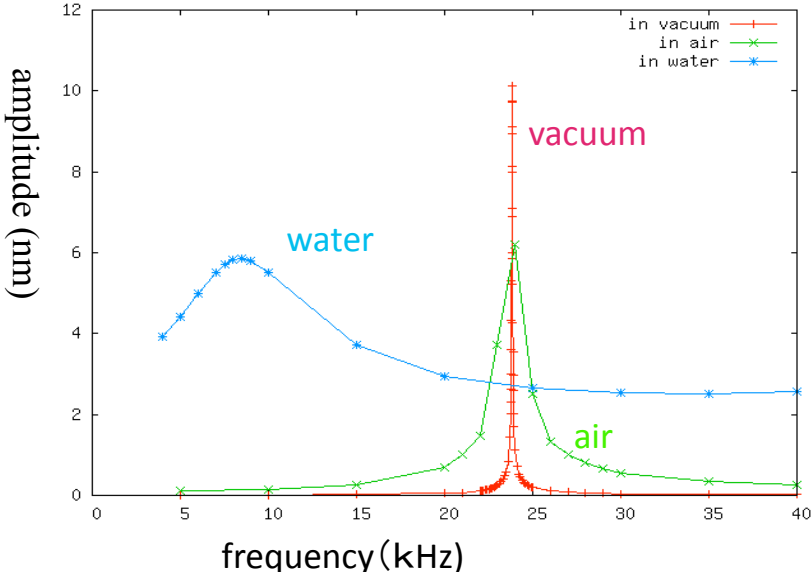
$\omega$  Is solved by FEM

Force felt by cantilever is  
Given by

$$F_s = \oint \left( P + \frac{\omega}{\text{Re}} \right) dl$$



# Calculation methods of the cantilever motion in liquids



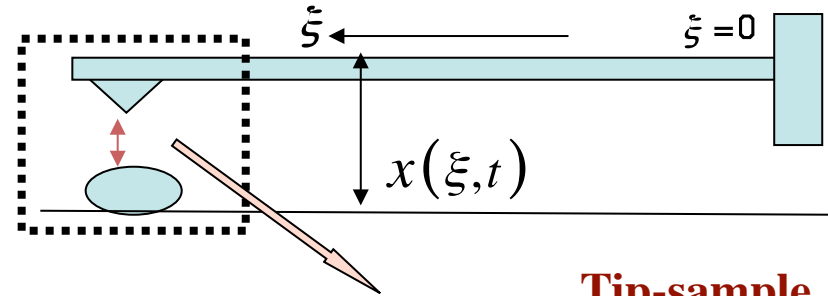
# Simulation of tapping mode AFM in liquid

$$x(\xi, t) \cong \sum_n x_n(t) \phi_n(\xi)$$

Projecting onto a certain mode

$$(1 + \kappa) \frac{d^2 x}{dt^2} + (\tilde{\gamma} + \gamma_{liq} + \gamma_{diss}) \frac{dx}{dt} + \omega_0^2 x = F_{driv}(t) + F_{TS}(x)$$

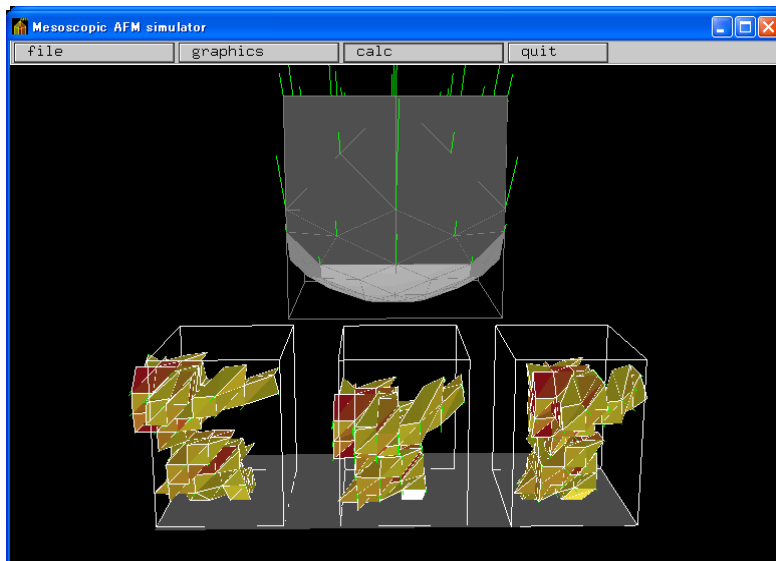
**Tip-sample force**



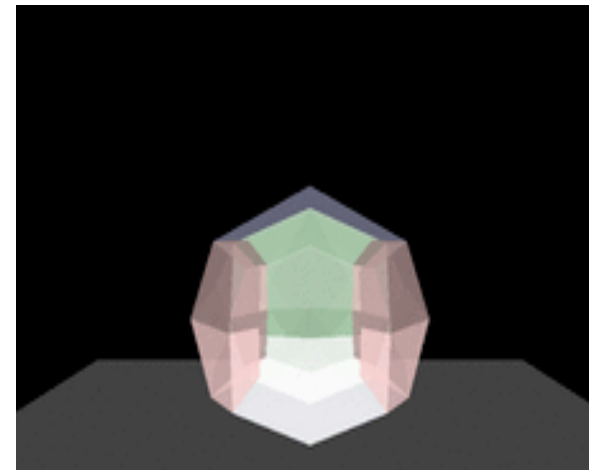
Coarse graining of protein molecules



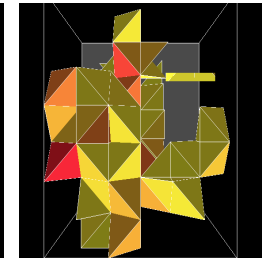
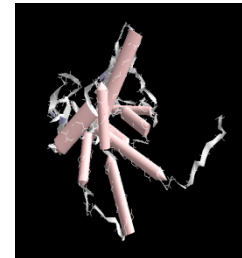
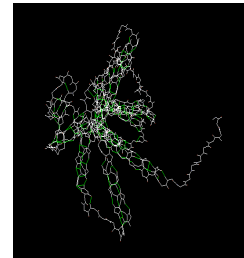
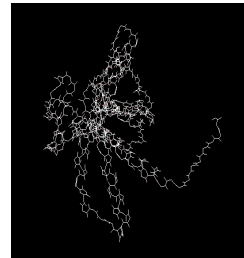
Visco-elastic model



Coarse graining of the sample

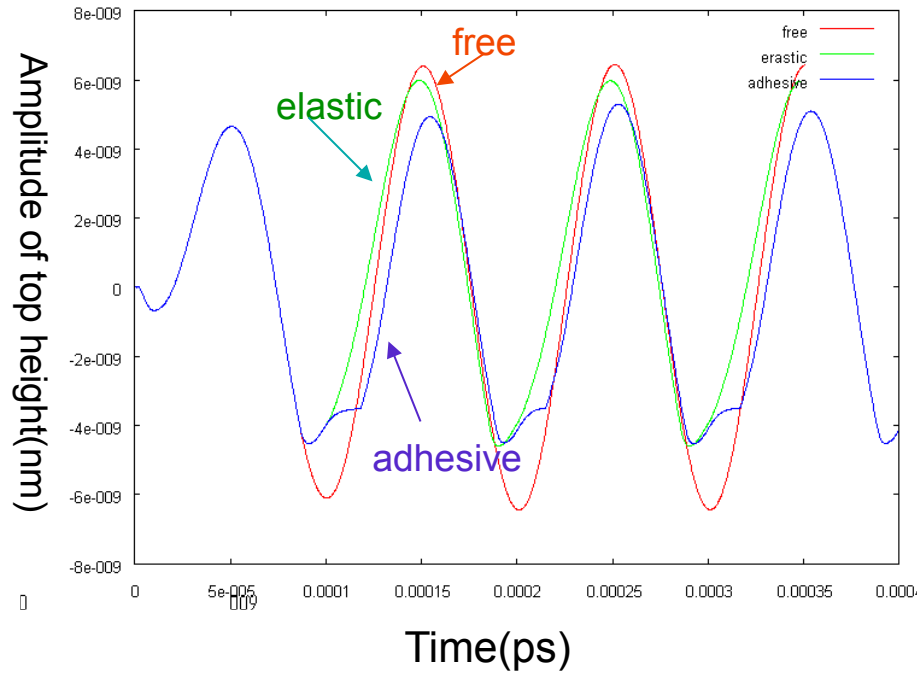


Tapping process

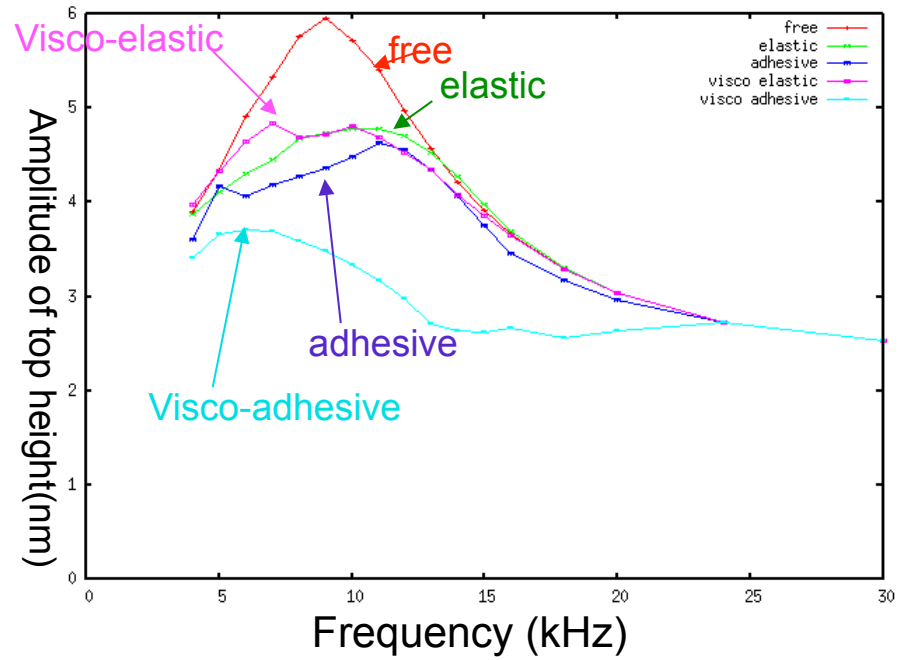


# Analyses of cantilever oscillation in the tapping mode AFM in water

Height of the cantilever top



Resonant curves for various collisions



elastic

$$f(h) = -k(h - h_{\text{touch}}) \quad h < h_{\text{touch}}$$

adhesive

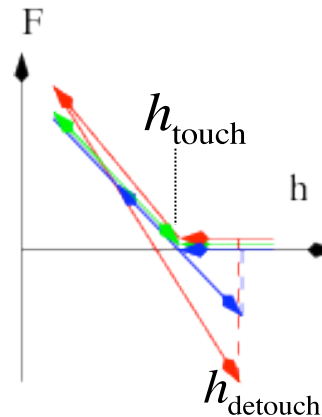
$$f(h) = -k(h - h_{\text{touch}}) \quad \begin{cases} h < h_{\text{touch}} \\ h < h_{\text{detach}} \end{cases}$$

Visco-elastic

$$f(h) = -k(h - h_{\text{touch}}) - \gamma v \quad h < h_{\text{touch}}$$

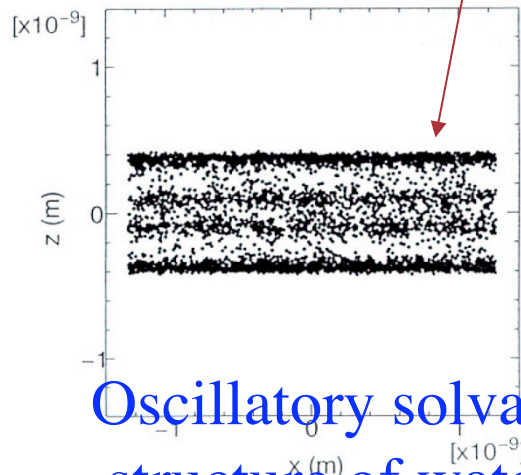
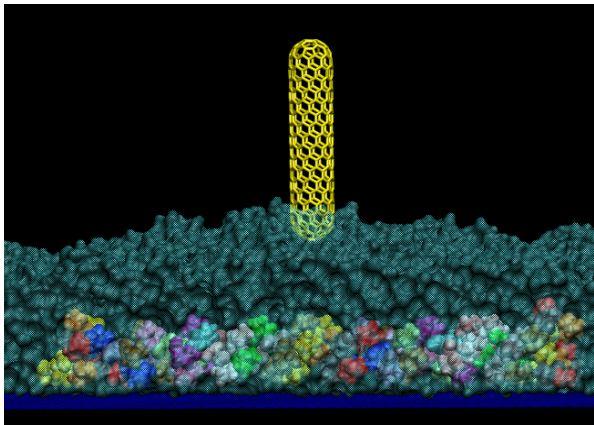
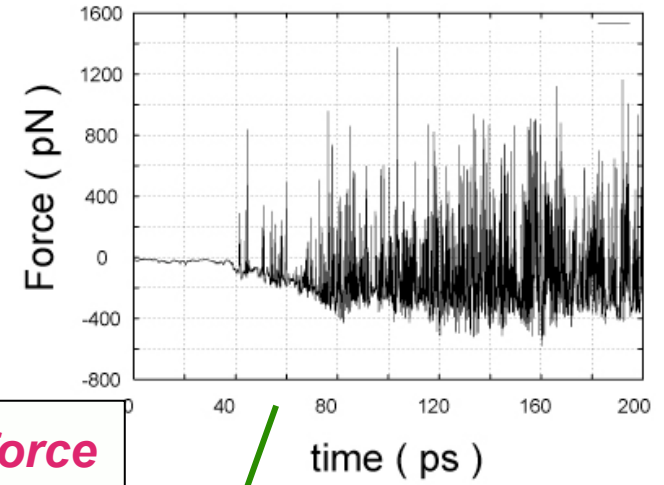
Visco-adhesive

$$f(h) = -k(h - h_{\text{touch}}) - \gamma v \quad \begin{cases} h < h_{\text{touch}} & v < 0 \\ h < h_{\text{detach}} & v > 0 \end{cases}$$



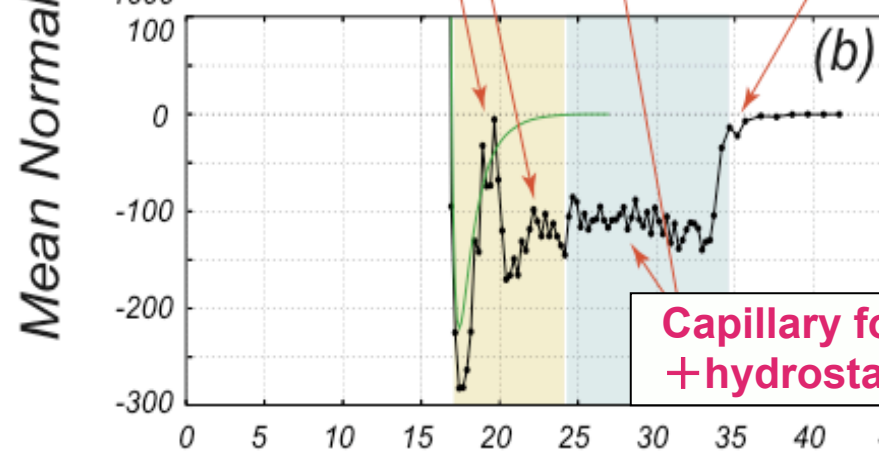
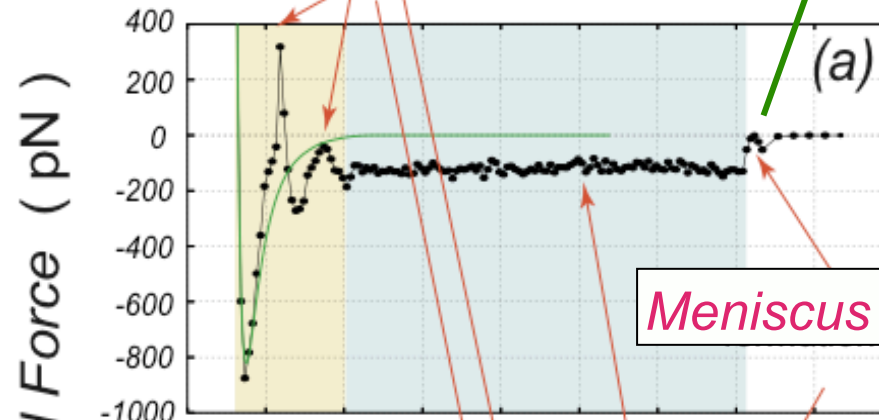
# Tip-sample interaction force Mediated by water

## AFM simulation by MD in liquids



Oscillatory solvation structure of water

Oscillatory solvation force



Tip Height

# Simulation of nc-AFM of mica in water-classical MD method

## *MD condition*

**Size of mica surface :** 36A×42A

**Tip model :** (10,0)CNT

**Potential of water molecules :** TIP3P

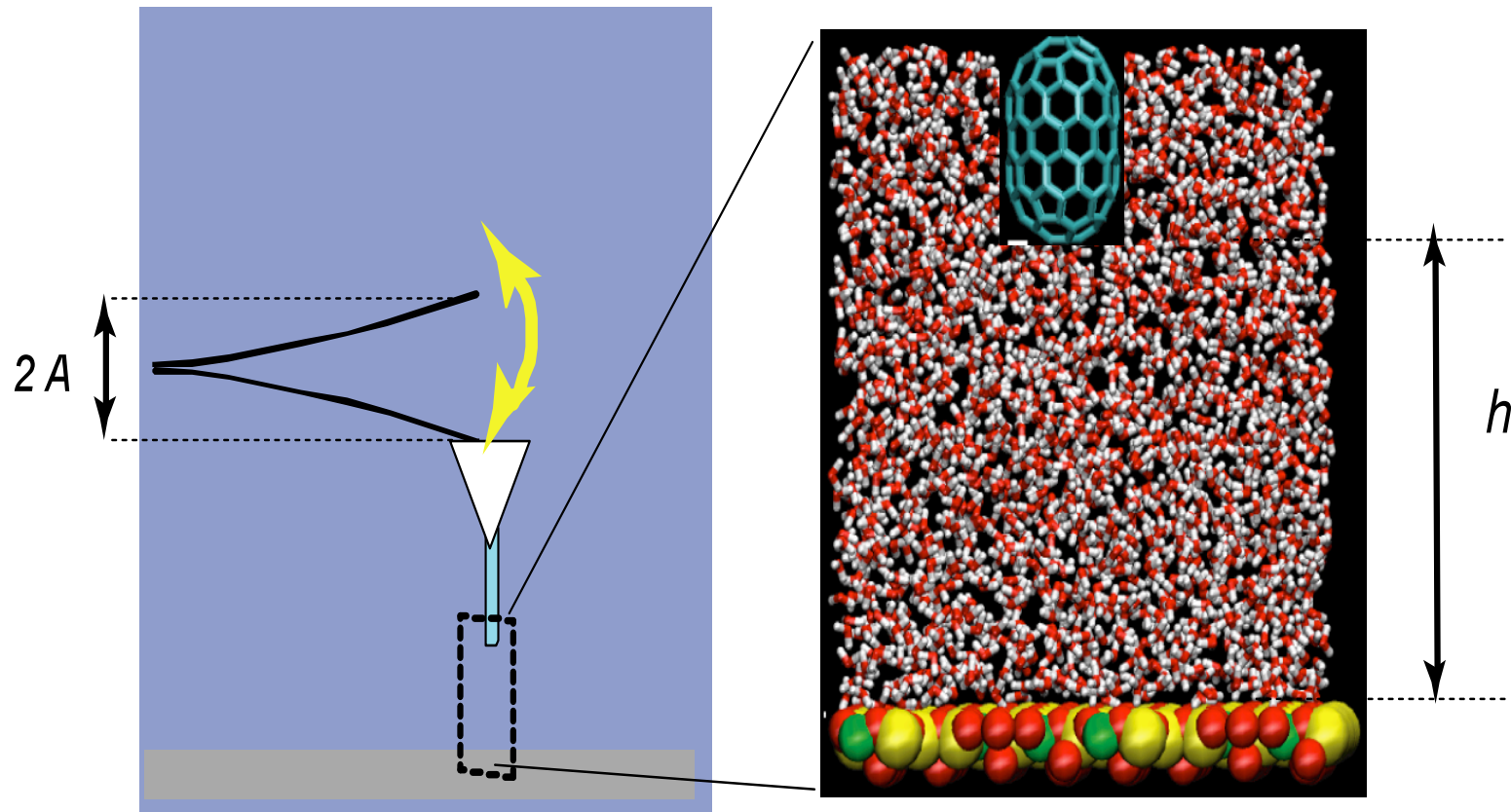
**Total number of atoms :** 6,338

**Force field:** CHARMM 22 + CLAY ( modified )

**Program:** NAMD2.5 and 2.6

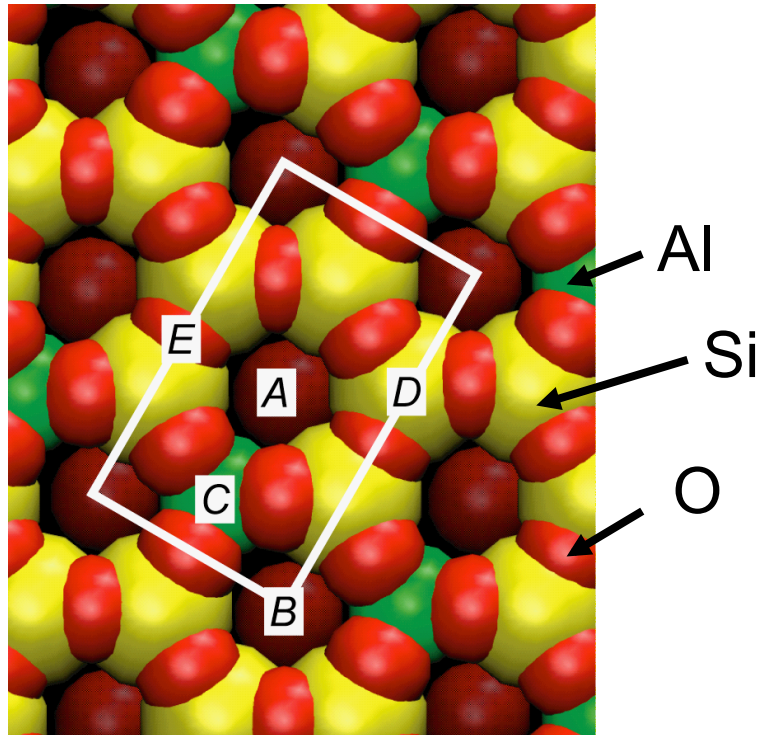
**Temperature:** 300 K

**Time mesh:** 2fs



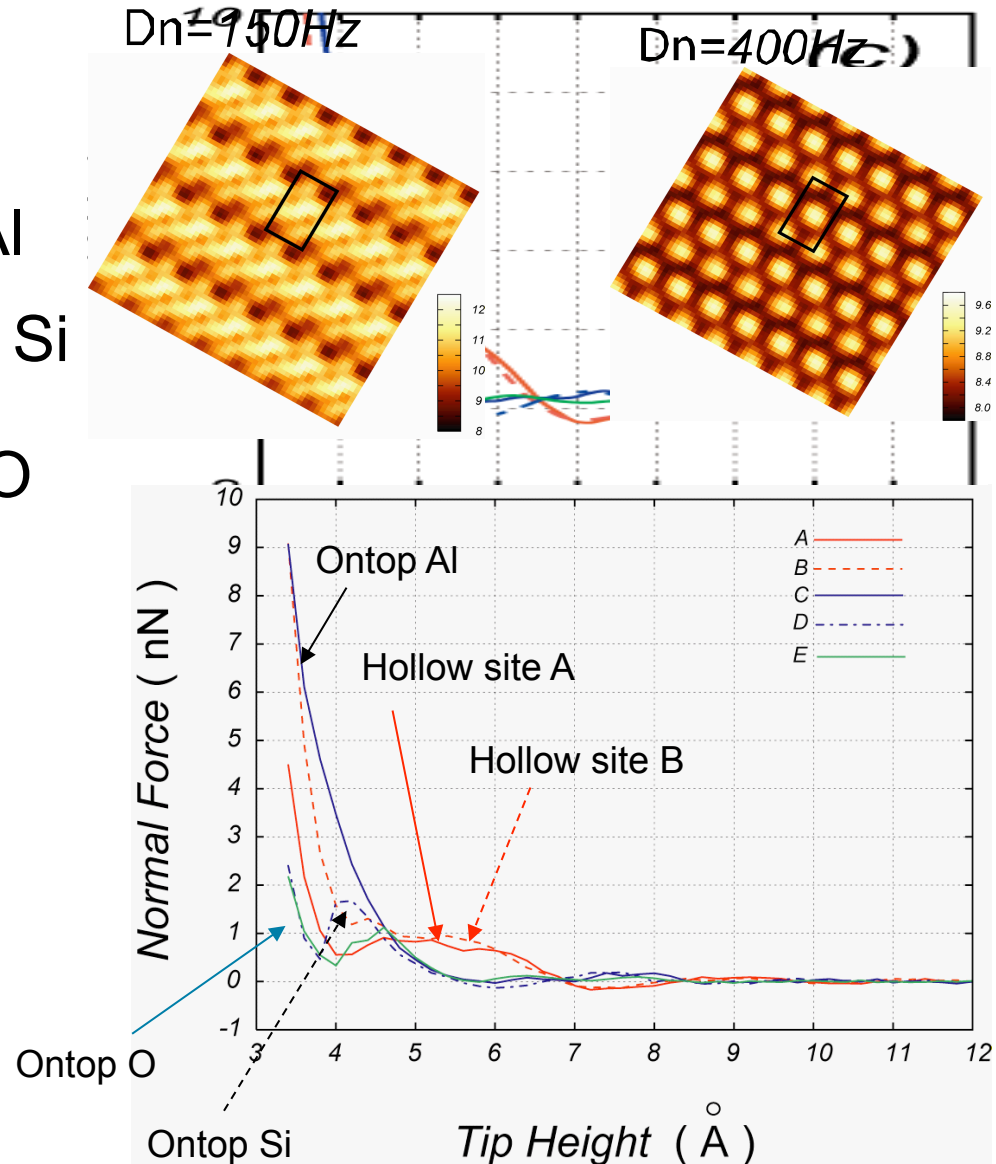


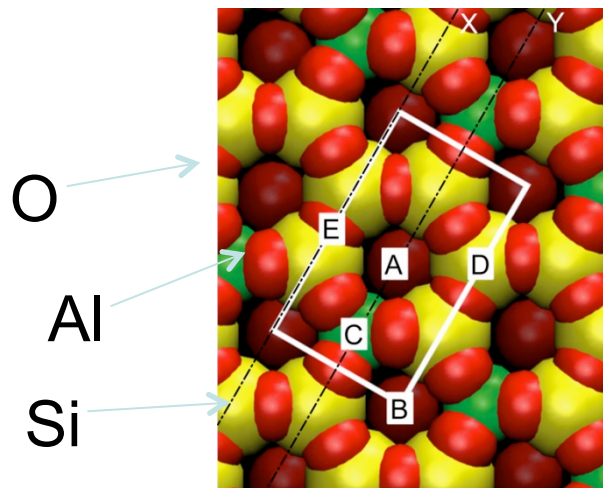
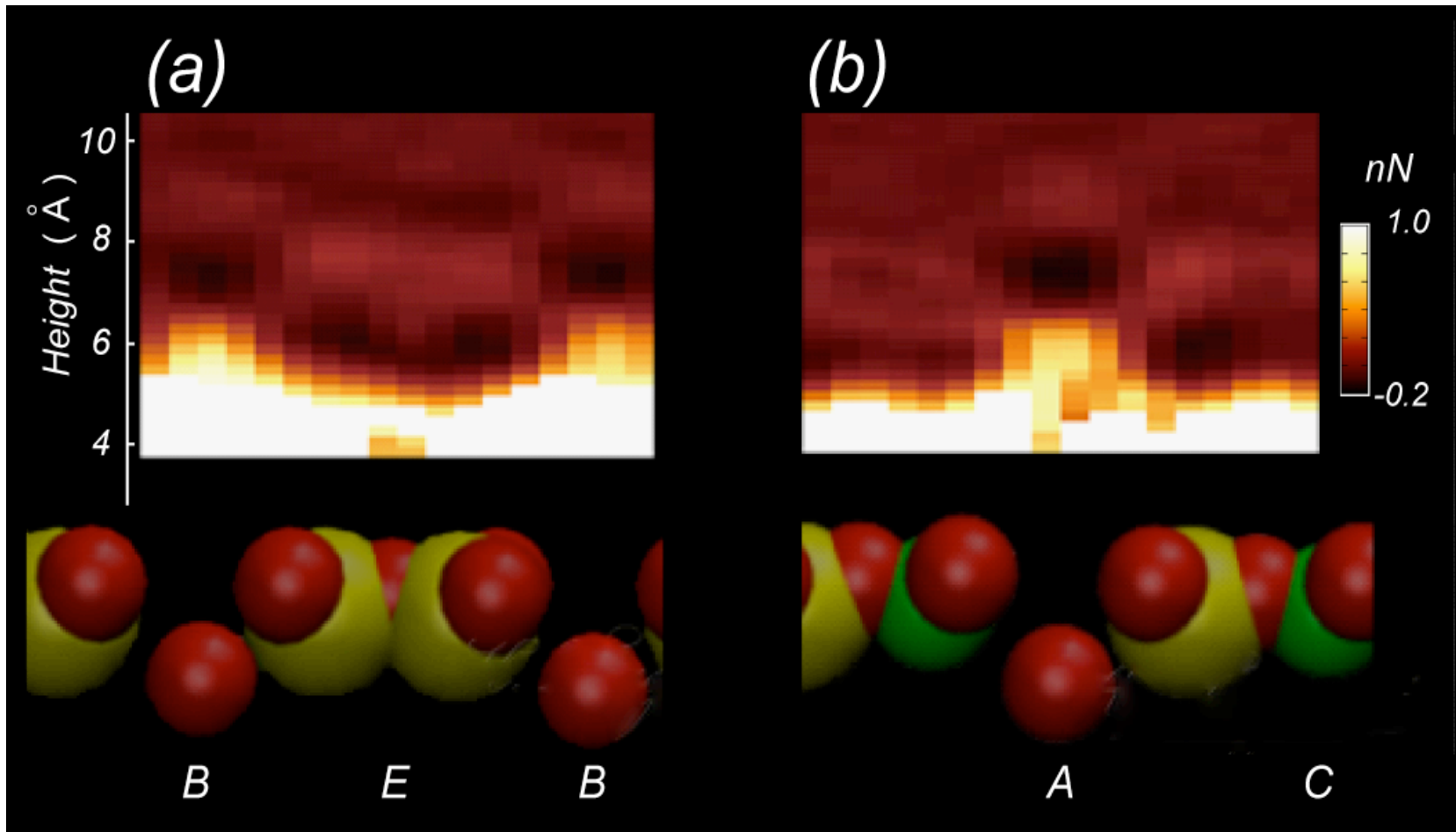
# Simulated dynamic AFM image of mica in water



Mica surface in water

- A, B: *ontop of hollow site*
- C: *ontop of Al atom*
- D: *ontop of Si atom*
- E: *ontop of O atom*



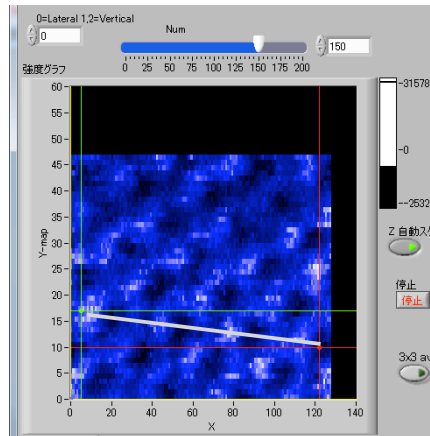
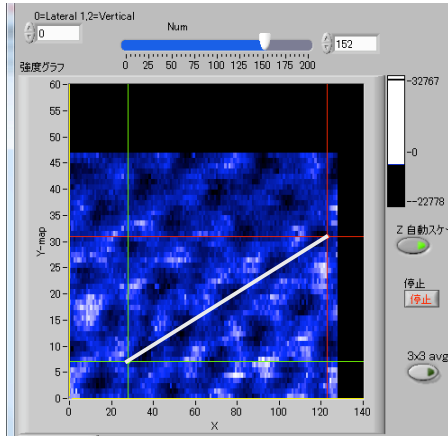


Cross section of 3D force map  
of mica surface in water  
(SWNT tip)

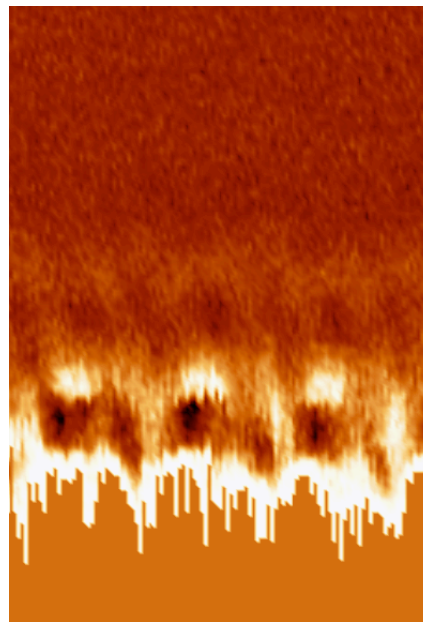
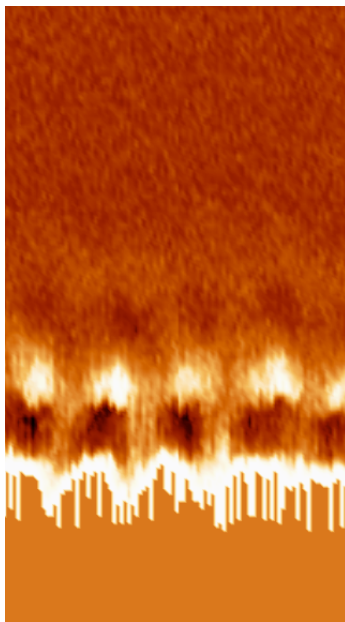
*A, B: ontop of hollow site C: ontop of Al atom  
D: ontop of Si atom E: ontop of O atom*

# Comparison between the theory and the experiment

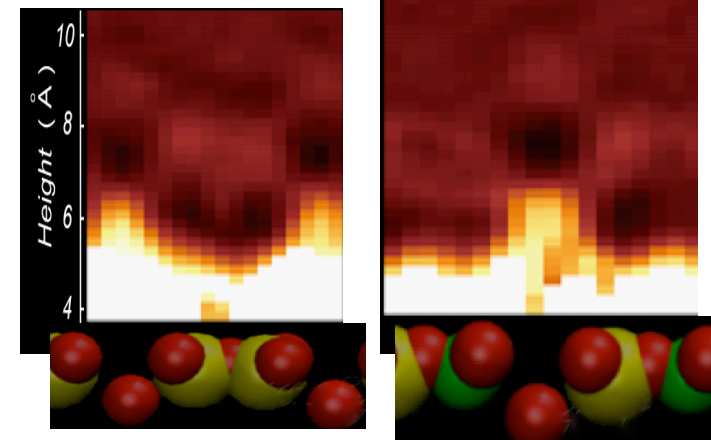
## Experiments H.Yamada, et al



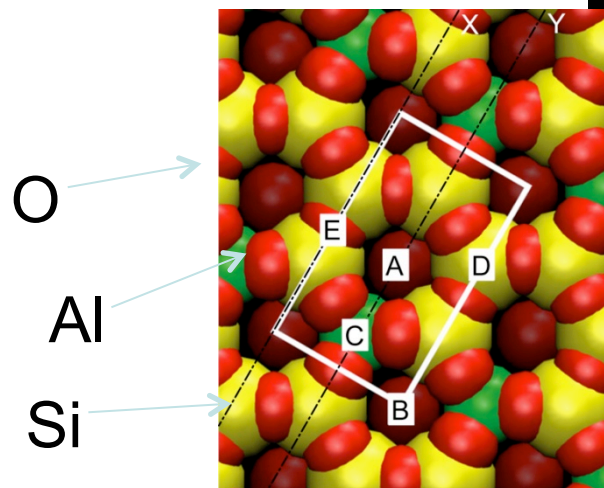
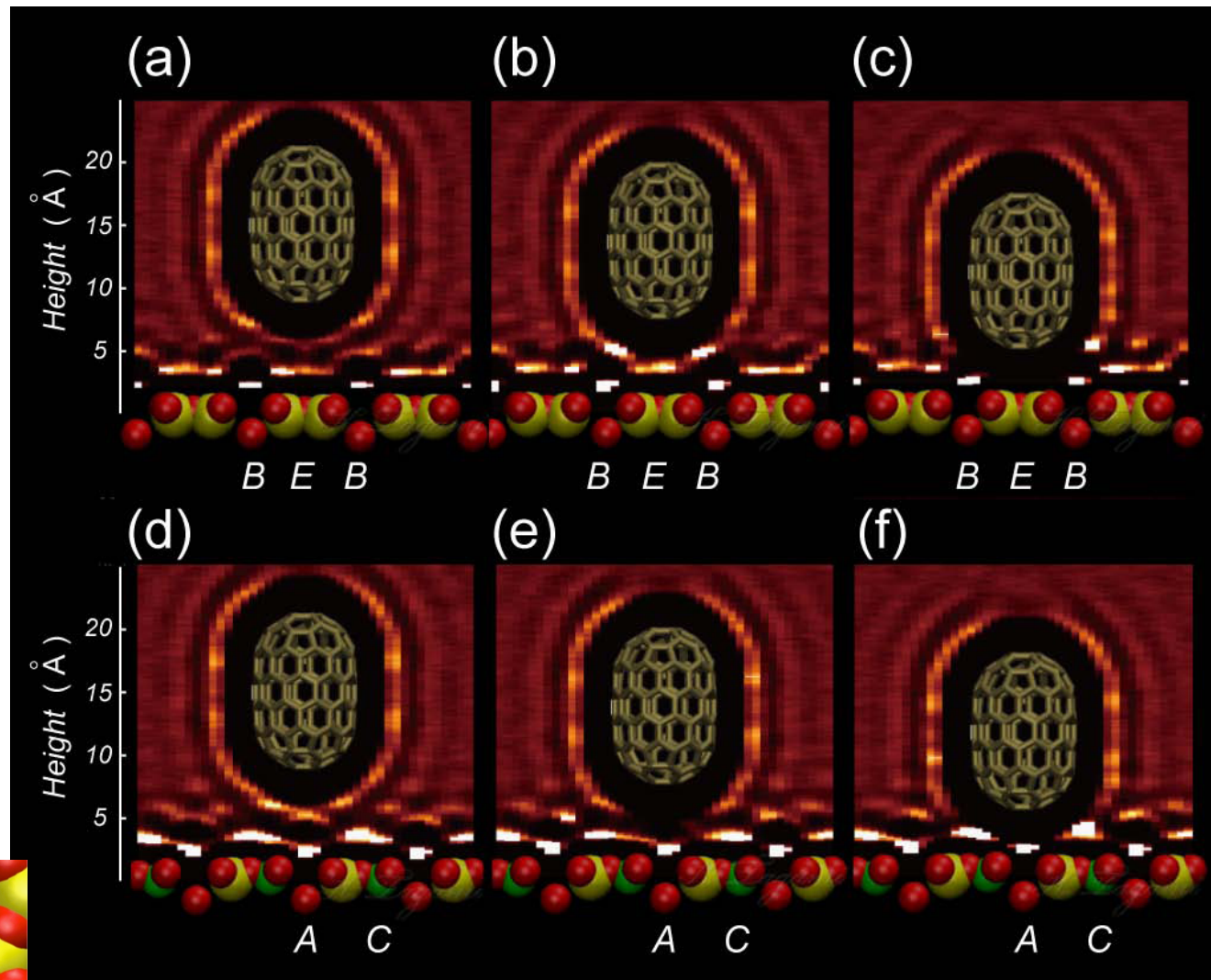
Results of theoretical simulation reproduced fairly well the experimental observation of nc-AFM images of mica surface in water.



## Theoretical Simulation



# Distribution function of water molecules



*A, B: ontop of hollow site C: ontop of Al atom  
D: ontop of Si atom E: ontop of O atom*

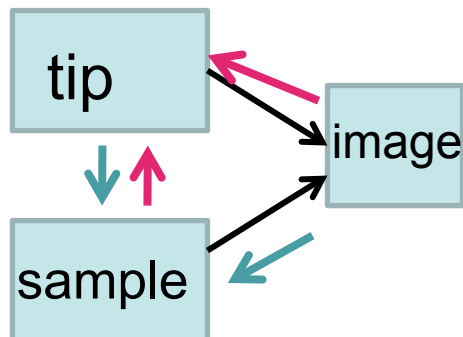
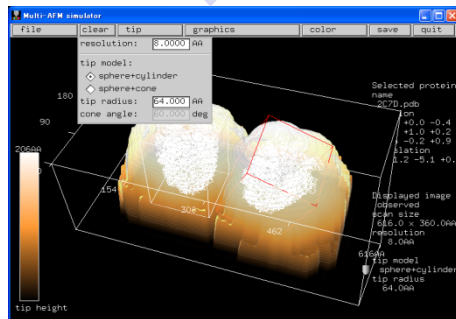
# Summary and outlook

- 1 Theoretical methods for STM such as Bardeen's Perturbation theory, NEGF method, RTM are reviewed are presented. Problems discussed are Effect of the tip, decorated tip, weak to strong interaction
- 2 Coherent and Dissipating Tunnelling in the STM systems are discussed including tunneling current distribution, features of the coherent current in the nano-structures, from the coherent to dissipative tunneling, zero bias anomaly
- 3 Theories of dynamic AFM in vacuum and in water are presented as derivation of the harmonic oscillator model, dissipation image, tip effects, cantilever oscillation, oscillatory hydration force, force mediated by water
- 4 Simulations of Nano-mechanics of protein molecules

# SPM Simulator Project

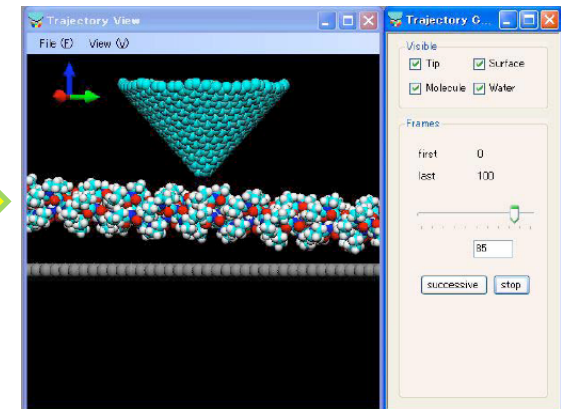
## 1. Rapid AFM simulator for tip-sample-image

Utilizing the geometrical relation reinforced by the continuum mechanics



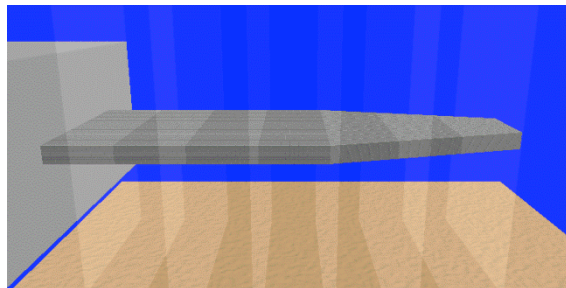
## 3. AFM image simulator for atoms/molecules/nanostructures

Using classical MD  
Molecular field  
3D-RISM



## 2. AFM simulator for soft materials in liquids

Cantilever oscillation in liquids  
Analyses of AFM for visco-elastic sample models



## 4. Quantum mechanical AFM/STM /KFM simulator

Using DFT, DFTB, DVX $\alpha$  method

